Homework 0: Math Review

CS 585, UMass Amherst, Fall 2018

Note

Wikipedia is a useful resource for basic probability and linear algebra.

Make a PDF file of your answers, and upload it to Gradescope by the end of Tues Sep 11. We will only accept PDF format.

1 Domain of a joint distribution

1.1

A and B are discrete random variables. A can take on one of 10 possible values. B can take on one of 32 possible values. (In other words, the size of domain(A) is 10, and the size of domain(B) is 32.) How many possible outcomes does the joint distribution $P(A, B)$ define probabilities for?

1.2

Say we have a sequence of n binary random variables $A_1, A_2, \ldots A_n$. How many possible outcomes does the joint distribution $P(A_1, A_2, \ldots A_n)$ define probabilities for?

2 Independence versus Basic Definitions

Say we have three random variables A and B and C. Note that we're using standard probability theory notation where $P(A, B) = P(B, A)$, which simply means the joint probability of both A and B occurring.

2.1

Which of the following statements is always true?

1. $P(A|B) = P(B|A)$
2. $P(A, B) = P(A|B)P(B)$
3. $P(A, B) = P(A)P(B)$
4. $P(A|B) = P(A)$
5. $P(A, B, C) = P(A)P(C)$
6. $P(A, B, C) = P(A)P(B)P(C)$

7. $P(A, B, C) = P(A)P(B|A)P(C|A, B)$
8. $P(A) = \sum_{b \in \text{domain}(B)} P(A, B = b)$
9. $P(A) = \sum_{b \in \text{domain}(B)} P(A|B = b)P(B = b)$
10. $\log(P(A)P(B)) = \log P(A) + \log P(B)$

2.2

Now assume that A, B, and C are all independent of each other. Which of the above statements are now true?

3 Logarithms

3.1 Log-probs

Let p be a probability, so it is bounded to $[0, 1]$ (between 0 and 1, inclusive). What is the range of possible values for $\log(p)$? Please be specific about open versus closed intervals.

3.2 Prob ratios

Let p and q both be probabilities. What is the range of possible values for p/q?

3.3 Log prob ratios

What is the range of possible values for $\log(p/q)$?

4 Linear algebra review

$\boldsymbol{x}$ is a 6-d real-valued vector (i.e., $\boldsymbol{x} \in \mathbb{R}^6$). $\boldsymbol{y}$ is another vector of the same dimensionality ($\boldsymbol{y} \in \mathbb{R}^6$). $\mathbf{W}_1$ is a 6×6 real-valued matrix, and $\mathbf{W}_2$ is a 12×6 real-valued matrix.

Answer the following questions. Feel free to look at online resources such as Wikipedia for help, and/or additionally test out your answers programmatically using libraries such as numpy.

1. what is the dimensionality of the element-wise product $\boldsymbol{x} * \boldsymbol{y}$?
2. what is the dimensionality of the dot product of $\boldsymbol{x}$ and $\boldsymbol{y}$ (i.e., $\boldsymbol{x} \cdot \boldsymbol{y}$, or $\boldsymbol{x}^\intercal \boldsymbol{y}$ in matrix notation)?
3. what is the dimensionality of the matrix-vector product $\mathbf{W}_1 \boldsymbol{x}$?
4. what is the dimensionality of $\mathbf{W}_2 \boldsymbol{y}$?
5. assume the magnitude of $\boldsymbol{x}$ is 1 (i.e., $\|\boldsymbol{x}\| = 1$). what is $\boldsymbol{x} \cdot \boldsymbol{x}$?
6. assume $\boldsymbol{x}$ and $\boldsymbol{y}$ are orthogonal, and $\|\boldsymbol{x}\| = \|\boldsymbol{y}\| = 1$. what is $\boldsymbol{x} \cdot \boldsymbol{y}$?