



# Conditional Probability Practice

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## Don't Panic

- Freakout on Piazza on NY / WV (4 districts = .1 points / 40)
- HW2 released in next few days
- Office hours huge success

## Today

- Conditional probability problems
  - Important for classification
  - You'll be estimating them in HW2

## Dice

A die is rolled twice

what is the probability that the sum of the faces is greater than 7, given that

- the first outcome was 4?
- the first outcome was greater than 4?
- the first outcome was a 1?
- the first outcome was less than 5?

## Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) =$
- $p(X_1 + X_2 > 7 | X_1 > 4) =$
- $p(X_1 + X_2 > 7 | X_1 = 1) =$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$

## Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1 + X_2 > 7 \wedge X_1 = 4)}{p(X_1 = 4)} =$
- $p(X_1 + X_2 > 7 | X_1 > 4) =$
- $p(X_1 + X_2 > 7 | X_1 = 1) =$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$

## Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1 + X_2 > 7 \wedge X_1 = 4)}{p(X_1 = 4)} = \frac{3/36}{1/6} = \frac{1}{2}$
- $p(X_1 + X_2 > 7 | X_1 > 4) =$
- $p(X_1 + X_2 > 7 | X_1 = 1) =$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$

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- $p(X_1 + X_2 > 7 | X_1 > 4) = \frac{p(X_1 + X_2 > 7 \wedge X_1 > 4)}{p(X_1 > 4)} = \frac{9/36}{1/3} = \frac{27}{36} = \frac{3}{4}$
- $p(X_1 + X_2 > 7 | X_1 = 1) =$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$

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- $p(X_1 + X_2 > 7 | X_1 = 1) = \frac{p(X_1 + X_2 > 7 \wedge X_1 = 1)}{p(X_1 = 1)} =$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$

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- $p(X_1 + X_2 > 7 | X_1 = 1) = \frac{p(X_1 + X_2 > 7 \wedge X_1 = 1)}{p(X_1 = 1)} = \frac{0}{1/6} = 0$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$

## Dice

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- $p(X_1 + X_2 > 7 | X_1 < 5) = \frac{p(X_1 + X_2 > 7 \wedge X_1 < 5)}{p(X_1 < 5)} =$

## Dice

- $P(X_1 + X_2 > 7 | X_1 = 4) = \frac{P(X_1 + X_2 > 7 \wedge X_1 = 4)}{P(X_1 = 4)} = \frac{3/36}{1/6} = \frac{1}{2}$
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- $P(X_1 + X_2 > 7 | X_1 = 1) = \frac{P(X_1 + X_2 > 7 \wedge X_1 = 1)}{P(X_1 = 1)} = \frac{0}{1/6} = 0$
- $P(X_1 + X_2 > 7 | X_1 < 5) = \frac{P(X_1 + X_2 > 7 \wedge X_1 < 5)}{P(X_1 < 5)} = \frac{6/36}{2/3} = \frac{18}{64} = \frac{1}{4}$

## Children

What is the probability a family of two children has two boys

- given that it has at least one boy?
- given that the first child is a boy?

## Children

- $P(X_1 = T, X_2 = T | X_1 = T \vee X_2 = T) =$
- $P(X_1 = T, X_2 = T | X_1 = T) =$

## Children

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## Children

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- $P(X_1 = T, X_2 = T | X_1 = T) =$

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- $P(X_1 = T, X_2 = T | X_1 = T) = \frac{P(X_1 = T, X_2 = T)}{P(X_1 = T)} = \frac{1/4}{1/2} = \frac{1}{2}$

## Conditional Probabilities

One coin in a collection of 65 has two heads. The rest are fair. If a coin, chosen at random from the lot and then tossed, turns up heads 6 times in a row, what is the probability that it is the two-headed coin?

## Conditional Probabilities

- Let  $C$  be the coin chose (T for fake)
- Let  $H$  be the number of heads out of six

$$P(C = T | H = 6) = \tag{1}$$

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$$P(C = T | H = 6) = \frac{P(C = T \wedge H = 6)}{P(H = 6)} = \frac{1/65}{1/65 + \frac{64}{65} \cdot \frac{1}{2^6}} = \quad (1)$$

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## Bayes Rule

There's a test for Boogie Woogie Fever (BWF). The probability of getting a positive test result given that you have BWF is 0.8, and the probability of getting a positive result given that you do not have BWF is 0.01. The overall incidence of BWF is 0.01.

1. What is the marginal probability of getting a positive test result?
2. What is the probability of having BWF given that you got a positive test result?

## Bayes Rule

Let  $D$  be the disease,  $T$  be the test

- $P(T = \top) =$
- $P(D = \top | T = \top) =$

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Let  $D$  be the disease,  $T$  be the test

- $P(T = \top) = \sum_{x=\top, \perp} P(T = \top, D = x) =$
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## Bayes Rule

Let  $D$  be the disease,  $T$  be the test

- $P(T = \top) = \sum_{x=\top, \perp} P(T = \top, D = x) = 0.01 \cdot .8 + .99 \cdot .01 =$
- $P(D = \top | T = \top) =$

## Bayes Rule

Let  $D$  be the disease,  $T$  be the test

- $P(T = \top) = \sum_{x=\top, \perp} P(T = \top, D = x) = 0.01 \cdot .8 + .99 \cdot .01 = 0.02$
- $P(D = \top | T = \top) =$

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- $P(D = \top | T = \top) = \frac{P(T = \top | D = \top)P(D = \top)}{P(T = \top)} = \frac{0.8 \cdot 0.01}{0.02} =$

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- $P(D = \top | T = \top) = \frac{P(T = \top | D = \top)P(D = \top)}{P(T = \top)} = \frac{0.8 \cdot 0.01}{0.02} = 0.4$