Logistic Regression

Introduction to Data Science Algorithms

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SLIDES ADAPTED FROM HINRICH SCHÜTZE

What are we talking about?

- Statistical classification: $p(y|x)$
- y is typically a Bernoulli or multinomial outcome
- Classification uses: ad placement, spam detection
- Building block of other machine learning methods

Logistic Regression: Definition

- Weight vector β_i
- Observations X_i
- "Bias" β_0 (like intercept in linear regression)

$$P(Y=0|X) = \frac{1}{1+\exp\left[\beta_0 + \sum_i \beta_i X_i\right]} \tag{1}$$

$$P(Y=1|X) = \frac{\exp\left[\beta_0 + \sum_i \beta_i X_i\right]}{1+\exp\left[\beta_0 + \sum_i \beta_i X_i\right]} \tag{2}$$

- For shorthand, we'll say that

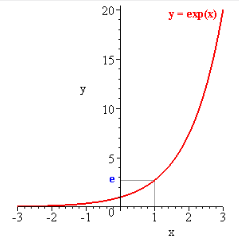
$$P(Y=0|X) = \sigma(-(\beta_0 + \sum_i \beta_i X_i)) \tag{3}$$

$$P(Y=1|X) = 1 - \sigma(-(\beta_0 + \sum_i \beta_i X_i)) \tag{4}$$

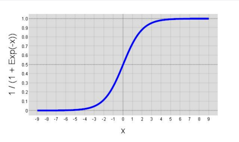
- Where $\sigma(z) = \frac{1}{1+exp[-z]}$

What’s this “exp” doing?

Exponential



Logistic



- $\exp[x]$ is shorthand for e^x
- e is a special number, about 2.71828
  - e^x is the limit of compound interest formula as compounds become infinitely small
  - It’s the function whose derivative is itself
- The “logistic” function is $\sigma(z) = \frac{1}{1+e^{-z}}$
- Looks like an “S”
- Always between 0 and 1.

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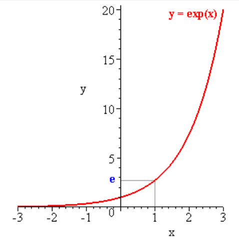
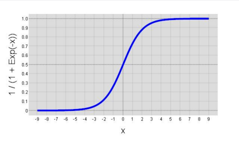


Logistic



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- e is a special number, about 2.71828
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- The "logistic" function is $\sigma(z) = \frac{1}{1+e^{-z}}$
- Looks like an "S"
- Always between 0 and 1.
  - Allows us to model probabilities
  - Different from **linear** regression

Logistic Regression Example

feature	coefficient	weight
bias	β_0	0.1
"viagra"	β_1	2.0
"mother"	β_2	−1.0
"work"	β_3	−0.5
"nigeria"	β_4	3.0

- What does $Y = 1$ mean?

Example 1: Empty Document?

$X = \{\}$

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Example 1: Empty Document?

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- $P(Y = 0) = \frac{1}{1+\exp[0.1]} =$
- $P(Y = 1) = \frac{\exp[0.1]}{1+\exp[0.1]} =$

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Example 1: Empty Document?

$X = \{\}$

- $P(Y = 0) = \frac{1}{1+\exp[0.1]} = 0.48$
- $P(Y = 1) = \frac{\exp[0.1]}{1+\exp[0.1]} = 0.52$
- Bias β_0 encodes the prior probability of a class

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Example 2

$X = \{\text{Mother}, \text{Nigeria}\}$

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Example 2

$X = \{\text{Mother}, \text{Nigeria}\}$

- $P(Y=0) = \frac{1}{1+\exp[0.1-1.0+3.0]} =$
- $P(Y=1) = \frac{\exp[0.1-1.0+3.0]}{1+\exp[0.1-1.0+3.0]} =$
- Include bias, and sum the other weights

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Example 2

$X = \{\text{Mother}, \text{Nigeria}\}$

- $P(Y=0) = \frac{1}{1+\exp[0.1-1.0+3.0]} = 0.11$
- $P(Y=1) = \frac{\exp[0.1-1.0+3.0]}{1+\exp[0.1-1.0+3.0]} = 0.88$
- Include bias, and sum the other weights

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Example 3

$X = \{\text{Mother}, \text{Work}, \text{Viagra}, \text{Mother}\}$

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- What does $Y = 1$ mean?

Example 3

$X = \{\text{Mother}, \text{Work}, \text{Viagra}, \text{Mother}\}$

- $P(Y = 0) = \frac{1}{1+\exp[0.1-1.0-0.5+2.0-1.0]} =$
- $P(Y = 1) = \frac{\exp[0.1-1.0-0.5+2.0-1.0]}{1+\exp[0.1-1.0-0.5+2.0-1.0]} =$
- Multiply feature presence by weight

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- What does $Y = 1$ mean?

Example 3

$X = \{\text{Mother}, \text{Work}, \text{Viagra}, \text{Mother}\}$

- $P(Y=0) = \frac{1}{1+\exp[0.1-1.0-0.5+2.0-1.0]} = 0.60$
- $P(Y=1) = \frac{\exp[0.1-1.0-0.5+2.0-1.0]}{1+\exp[0.1-1.0-0.5+2.0-1.0]} = 0.30$
- Multiply feature presence by weight