Talks and Projects For the Ramsey Gang

Every page has a new project.
Some of the pages will have slides from a talk about the topic.
Some of the pages will have a recording of a talk about the topic.
Some will have a pointer to notes about the topic.
Some will have project ideas.
Some will be written before I talk about it to you.

1 Rectangle Free Colorings of Grids

Slides:
`https://www.cs.umd.edu/~gasarch/COURSES/250/S24/slides/COMB/gridtalk.pdf`
Recording: There is no recording.

Project Ideas:
I proved that a grid is 2-colorable iff it does NOT have 5×5 or 3×7 or 7×3 inside it.

1. Obtain this result with a SAT Solver or a program rather than by clever math.

2. Obtain the obstruction set for 3-coloring. (This is known.)

3. Obtain the obstruction set for 4-coloring. (This is known.)

4. Obtain the obstruction set for 5-coloring. (This is NOT known.)

5. Instead of rectangles look at squares. (It is known that for every c-coloring of some large enough grid there is a mono square. For $c = 2$ its known. Beyond that, unknown. Obstructs sets are unknown.)

6. Instead of rectangles look at right triangles. You may also demand they be similar to a particular right triangle with natural number sides (e.g., 3-4-5). I do not know whats known here. See later project about Euclidean Ramsey Theory.

2 Approximation of Reals by Rationals

The talk and slides are on the question:

Is $\{a + b\sqrt{2} : a, b, \in \mathbb{Z}\}$ **Dense?**

However it leads into a project about approximating reals.

Slides:

`https://www.cs.umd.edu/~gasarch/COURSES/250/S24/slides/COMB/denseab.pdf`

Recording:

`https://www.cs.umd.edu/~gasarch/RAMSEYGANG/density.mp4`

Project Ideas

1. In the slides we proved the following towards the end (I changed the names of the variables because I need to for my next point).

   Theorem 2.1 *Let* $\gamma \in \mathbb{I}$ *(irrational). Then*

   $$(\exists^\infty q \in \mathbb{N})(\exists p \in \mathbb{Z})\left[\left|\gamma - \frac{p}{q}\right| < \frac{1}{q^2}.\right]$$

   Can we do better? For example, might the following be true:

   $$(\exists^\infty q \in \mathbb{N})(\exists p \in \mathbb{Z})[|\gamma - \frac{p}{q}| < \frac{1}{q^3}.$$

   Or perhaps a more modest goal:

   $$(\exists^\infty q \in \mathbb{N})(\exists p \in \mathbb{Z})[|\gamma - \frac{p}{q}| < \frac{1}{10q^2}.$$

   NO, this cannot be achieved. In the document.

   `https://www.cs.umd.edu/~gasarch/HURWITZ/hurwitz.pdf`

   I show (this was already known a long time ago) that for $\gamma = \frac{1+\sqrt{5}}{2}$

   $$(\exists N \in \mathbb{N})(\forall q \geq \mathbb{N})(\forall p \in \mathbb{N})[|\gamma - \frac{p}{q}| \geq \frac{1}{\sqrt{5}q^2}.$$

   In this project you will see how well irrationals can be approximated .

Details on this project on the next page.

3 TO DO List for Approx Irrationals Project

I will give three methods to generate a sequence of approximations to an irrational ζ. I will then say what you DO with the sequence, regardless of where it came from.

1) Continued Fractions Method READ up on continued fractions. WRITE a program that will, given an irrational ζ (you need to figure out how you can be GIVEN an irrational) generate the continued fraction expansion for ζ, up to (say) 100. Use this to generate a sequence of rational approximations for ζ

$$\frac{p_1}{q_1}, \frac{p_2}{q_2}, \ldots, \frac{p_{100}}{q_{100}}.$$

2) Try lots of pairs $\{p, q\}$

Let $a \in \mathbb{N}$ such that $a < \zeta < a + 1$. We know that the only rationals worth considering are $\frac{p}{q}$ such that $a < \frac{p}{q} < a + 1$.

Write a program that looks at every $\{p, q\}$ with $1 \le p, q \le 1000$ where p, q are rel prime and $a < \frac{p}{q} < a + 1$. For each of these $\{p, q\}$ we have the rational $\frac{p}{q}$. These are our

$$\frac{p_1}{q_1}, \frac{p_2}{q_2}, \ldots, \frac{p_{100}}{q_{100}}.$$

(There may be more or less than 100. If its to few then change 1000 to (say) 2000.)

3) Use Dirichlet's Proof

READ the following sides from slide 89 on.

`https://www.cs.umd.edu/~gasarch/COURSES/250/S24/slides/COMB/denseab.pdf`

The proof that you can get a good approx gives you a way to find it. Use this to get

$$\frac{p_1}{q_1}, \frac{p_2}{q_2}, \ldots, \frac{p_{100}}{q_{100}}.$$

ONCE you have the sequence, what do to with it?

Assume you got the sequence

$$\frac{p_1}{q_1}, \frac{p_2}{q_2}, \ldots, \frac{p_{100}}{q_{100}}$$

from one of the three methods above.

1. For $1 \le i \le 100$ find c_i such that $|\zeta - \frac{p_i}{q_i}| = \frac{c_i}{q_i^2}$.
2. Sort the $\frac{p_1}{q_1}, \frac{p_2}{q_2}, \ldots, \frac{p_{100}}{q_{100}}$ based on the c_i. Put this in a table. Note if c_i's are inc, dec, or about the same.
3. If the c_i's are decreasing then try replacing q_i^2 with q_i^3.
4. Look at the data and see what you think.

4 Small Ramsey Numbers

Slides:

https://www.cs.umd.edu/~gasarch/COURSES/250/S24/slides/COMB/smallramseytalk.pdf

Recording:

https://www.cs.umd.edu/~gasarch/RAMSEYGANG/smallramsey.mp4

Project Ideas:

1. The proof that $R(a, b)$ exists actually gave you an algorithm to find the RED K_a or BLUE K_b: Look for a high RED degree or high BLUE degree and look at the RED neighbors or the BLUE neighbors. (And then you will need to recurse.) CODE THIS UP and try it out on random graphs.

2. Random Ramsey Numbers. We use $RR(4)$ as an example. It is know that every 2-coloring of the edges of K_{18} has a mono K_4. It is know that there exists a 2-coloring of the edges of K_{17} with no mono K_4. But what if you randomly color the edges of K_{17} a million times? Will you find a coloring with no mono K_4 (I know from last years projects that you will not.) Look at K_{16}, K_{15}, etc until you get (a) SOME coloring has a no mono K_4, (b) about half of them have no mono K_4, (c) other fractions. SAME project for K_3, K_5. Graph the fraction, so you can have a graph where as n goes from 4 to 18 see how the fraction of colorings that have a mono K_4 goes down.

   The above assumed that the random graphs have prob of RED is 1/2 and of BLUE is 1/2. You can vary that. What if RED is 3/4 and BLUE is 1/4. Then you may need to go much lower to find a 2-coloring with no mono K_4.

   You might want to use the algorithm in the first project to FIND the monochromatic graphs. I do not know if that will work since failure IS option.

3. Some of the colorings that had no large mono cliques came from number theory. This did not work for, say, $R(5)$. However, one could still look at graphs of that type and see if they have large mono cliques.

5 Ramsey Multiplicity

Slides:

https://www.cs.umd.edu/~gasarch/COURSES/752/S22/slides/2tritalk.pdf

Link to Recording and Passcode needed:
I have it twice- right now both work but just in case that changes
CLICK HERE FOR RECORDING
CLICK HERE FOR RECORDING
Passcode: YIj?2zbg

Project Ideas

1. Note the following contrast.

   (a) Computer Searches showed that every 2-coloring of the edges of K_{18} has 9 mono K_4's.

   (b) I showed using MATH that every 2-coloring of K_{19} has 2 mono K_4"s. By the first result we know this result is NOT the best possible.

   Try to give a MATH proof of better results. Perhaps a combination of MATH and mild Computer Work.

2. Try to prove an asymmetric theorem, something like For every 2-coloring of K_n there is either a mono K_3's or b mono K_4's. I will talk about some ideas on this when we meet. (This has already been done, but asymptotically. I would want to see some actual numbers.)

3. Use the proofs I gave to write a PROGRAM that will, given a 2-coloring of the edges of K_n, FIND the mono K_n's

4. Find the RANDOMIZED version of this problem. For example, for a random 2-coloring of K_n how many mono triangles do you get? I think its lots more than $\frac{n^3}{24}$.

6 The Probabilistic Method

The lecture first revisited UPPER bounds on $R(a, b)$ which is at the end of the Small Ramsey Numbers slides, and then proved LOWER bounds on $R(a, b)$ using The Prob Method.

The slides are here:

`https://www.cs.umd.edu/~gasarch/COURSES/752/S22/slides/probmethodtalk.pdf`

The recording is here:

`https://umd.zoom.us/rec/share/izZB3HF9olto2sHYDwpHb-NTveeFesjeLMFeWN4IROGyXhAOFoOVFDs6RGWP6YslHkeUgUj`

Passcode: 0#9PYWG$

Project Ideas Take *any* of the Theorems or Algorithms that used the prob method, and see what happens if you convert it to a real algorithm that flips coins. How well would it do?