

BILL, RECORD LECTURE!!!!

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Rado's Thm

Exposition by William Gasarch

January 23, 2025

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What about other equations?

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(We can modify the proof to get a d-mono sol.)

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Thm $x + y = z$ is regular. (Can also show d-regular.)

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Take $x = y = z = 1$. Or any $x = y = z$.

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So get $x = a \quad x = a + 5d \quad y = a + 3d$.

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What is it about

$$2w + 3x = 5y$$

that made all of the a 's drop out? Discuss.

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We won't prove this but you have seen most of the ideas needed to prove it.

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$$2W + 3X = 5Y + Z$$

$$\text{We'll take } W = 2, X = 4, Y = 3, Z = 1$$

$$\text{So take } w = a + 2d \quad x = a + 4d \quad y = a + 3d \quad z = d$$

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How We Get What We Want

Want

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We won't prove. You have seen most of the ideas needed to prove it.

An Equation Where Rado Fails

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Case $e_3 < e_1, e_2$

Similar to $e_2 < e_1, 3_3$.

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$3b \equiv 0 \pmod{5}$ implies $b \equiv 0 \pmod{5}$. Contradiction.

Case $e_1 = e_3 < e_2$ and $e_2 = e_3 < e_1$

Similar to $e_1 = e_2 < e_3$.

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Take this mod 5 to get $3b \equiv 4b$ so $b \equiv 0 \pmod{5}$ Contradiction.

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5 is the lowest such prime.

Rado's Thm (Other Half of it)

Thm Let $a_1, \dots, a_k \in \mathbb{Z}$ be st no subset of the a_i 's sums to 0.

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Research Question

1. For $x + 2y = 4z$ what about 4-coloring? 3-coloring? 2-coloring?
2. More generally one can take an equation where no sum of the coefficients is 0 and look at colorings with a small number of colors.

Full Rado

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(For most equations with the coefficients sum to 0 you actually get d-regular.)

Misc

Research Questions

(Some is known about some of these.)

Prove or disprove that the equations below are regular.

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(Some is known about some of these.)

Prove or disprove that the equations below are regular.

1. $\sum_{i=1}^n a_i x_i = A$ for some A .
2. Higher degree equations (seems hard).

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(This equation has certain properties that make it work, so there is really a more general thm here.) <http://fourier.math.uoc.gr/~ergodic/Slides/Host.pdf>

[//fourier.math.uoc.gr/~ergodic/Slides/Host.pdf](http://fourier.math.uoc.gr/~ergodic/Slides/Host.pdf)

$x^2 + y^2 = z^2$ Result by Heule&Kullmann&Marek

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- 3) (Might be Hard) Obtain a human-readable proof with perhaps a much bigger N , but which can be generalized to $c = 3$ and beyond.