BILL, RECORD LECTURE!!!!

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Application! Restricting Domains To Stop Being Onto

Exposition by William Gasarch

February 6, 2025

Credit Where Credit Is Due

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This will not be our concern.

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What About $\mathbb{Z}[x]$ and $\mathbb{Z}[x, y]$

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In either case $f \colon \mathbb{D} \times \mathbb{D}$ is NOT onto.

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When Can We Restrict Domain So f Not Onto?

Question For which $f: \mathbb{Z} \to \mathbb{Z}$ is there
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Good Question For which $f \colon \mathbb{Z} \to \mathbb{Z}$ is there
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That wasn't stupid, but it was easy.

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Answer on next page.

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Given f, we will use Ramsey's Theorem 3 times.

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Why 3? We will discuss that later.

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Will now look at f restricted to $(x,y) \in H_1 \times H_1$ with $x < y$.

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Second Use of Ramsey

Define $\mathrm{COL}_2 \colon \binom{H_1}{2} \to [4]$

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If $\mathrm{COL}_2(H_2) = \textbf{\textcolor{red}{R}}$ then $(\forall x, y \in H_1, x < y)[f(x, y) \notin \{0, 1, 2\}]$.

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If $\mathrm{COL}_2(H_2) = \mathbf{R}$ then $(\forall x, y \in H_1, x < y)[f(x, y) \notin \{0, 1, 2\}]$.

Not done yet. On H_2 we control $f(x, x)$, $f(x, y)$ with $x < y$.

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Not done yet. On H_2 we control $f(x, x)$, $f(x, y)$ with $x < y$.

Now look at f restricted to $(x, y) \in H_1 \times H_1$ with $x > y$.

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We show that f on $H_3 \times H_3$ is not onto.

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One of the four colors is not here. Which one?

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if color 0 then 0 not in the image, so NOT onto.

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if color 0 then 0 not in the image, so NOT onto.

if color 1 then 1 not in the image, so NOT onto.

if color 2 then 2 not in the image, so NOT onto.

if color **R** then image is subset of $\{0, 1, 2\}$, so NOT onto.

f **Why** [**4**]?

f Why [4]?

Why four colors?

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4 is 1 more than the number possible types of inputs.

We will discuss this more after we do the Thin Set Theorem for $f(x, y, z)$.