Infinite Ramsey Theorem For Graphs

Exposition by William Gasarch

December 8, 2024

Notation

Let $a, n \in \mathbb{N}$. Let A be a set. A can be finite or infinite.

Notation

Let $a, n \in \mathbb{N}$. Let A be a set. A can be finite or infinite.

1. $\mathbb{N}$ is the naturals which are $\{1, 2, 3, \ldots\}$.

Notation

Let $a, n \in \mathbb{N}$. Let A be a set. A can be finite or infinite.

1. $\mathbb{N}$ is the naturals which are $\{1, 2, 3, \ldots\}$.
2. $[n] = \{1, \ldots, n\}$.

Notation

Let $a, n \in \mathbb{N}$. Let A be a set. A can be finite or infinite.

1. $\mathbb{N}$ is the naturals which are $\{1, 2, 3, \ldots\}$.
2. $[n] = \{1, \ldots, n\}$.
3. 2^A is the powerset of A.

Notation

Let $a, n \in \mathbb{N}$. Let A be a set. A can be finite or infinite.

1. $\mathbb{N}$ is the naturals which are $\{1, 2, 3, \ldots\}$.
2. $[n] = \{1, \ldots, n\}$.
3. 2^A is the powerset of A.
4. $\binom{A}{a}$ is the set of all a-sized subsets of A.

Notation

Let $a, n \in \mathbb{N}$. Let A be a set. A can be finite or infinite.

1. $\mathbb{N}$ is the naturals which are $\{1, 2, 3, \ldots\}$.
2. $[n] = \{1, \ldots, n\}$.
3. 2^A is the powerset of A.
4. $\binom{A}{a}$ is the set of all a-sized subsets of A.

Let $\mathrm{COL}\colon \binom{A}{2} \to [2]$. A set $H \subseteq A$ is **homogenous** if COL restricted to $\binom{H}{2}$ is constant. (From now on **homog**.)

Notation

Let $a, n \in \mathbb{N}$. Let A be a set. A can be finite or infinite.

1. $\mathbb{N}$ is the naturals which are $\{1, 2, 3, \ldots\}$.
2. $[n] = \{1, \ldots, n\}$.
3. 2^A is the powerset of A.
4. $\binom{A}{a}$ is the set of all a-sized subsets of A.

Let $\mathrm{COL}\colon \binom{A}{2} \to [2]$. A set $H \subseteq A$ is **homogenous** if COL restricted to $\binom{H}{2}$ is constant. (From now on **homog**.)

Party Definition There is a party. All the guests are members of A. Each pair either knows each other or does not know each other.

Notation

Let $a, n \in \mathbb{N}$. Let A be a set. A can be finite or infinite.

1. $\mathbb{N}$ is the naturals which are $\{1, 2, 3, \ldots\}$.
2. $[n] = \{1, \ldots, n\}$.
3. 2^A is the powerset of A.
4. $\binom{A}{a}$ is the set of all a-sized subsets of A.

Let $\mathrm{COL}\colon \binom{A}{2} \to [2]$. A set $H \subseteq A$ is **homogenous** if COL restricted to $\binom{H}{2}$ is constant. (From now on **homog**.)

Party Definition There is a party. All the guests are members of A. Each pair either knows each other or does not know each other.
$H \subseteq A$ is a **homog** if either
(a) every pair of elements of H knows each other, or
(b) every pair of elements of H does not knows each other.

The Infinite Ramsey Theorem

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ there exists an **infinite** homog set.

The Infinite Ramsey Theorem

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ there exists an **infinite** homog set.

We will, given $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$, form

The Infinite Ramsey Theorem

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ there exists an **infinite** homog set.

We will, given $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$, form

- An infinite subset $X \subseteq \mathbb{N}$.

The Infinite Ramsey Theorem

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ there exists an **infinite** homog set.

We will, given $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$, form

- An infinite subset $X \subseteq \mathbb{N}$.
- A 2-coloring of X

The Infinite Ramsey Theorem

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \rightarrow [2]$ there exists an **infinite** homog set.

We will, given $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \rightarrow [2]$, form

- An infinite subset $X \subseteq \mathbb{N}$.
- A 2-coloring of X

We do some an example of the first few steps of the constuction.

The Infinite Ramsey Theorem

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \rightarrow [2]$ there exists an **infinite** homog set.

We will, given $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \rightarrow [2]$, form

- An infinite subset $X \subseteq \mathbb{N}$.
- A 2-coloring of X

We do some an example of the first few steps of the constuction. My apologies to the math majors who are not used to seeing examples.

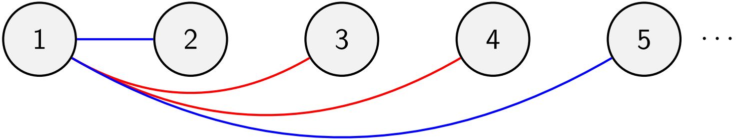
Examples of The First Few Steps of The Construction

First Step of Our Construction

Look at 1 and all of the edges coming out of it:

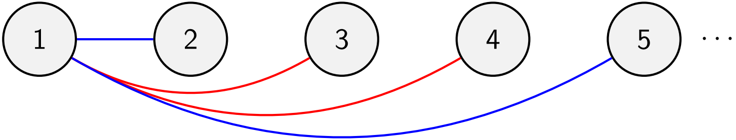
First Step of Our Construction

Look at 1 and all of the edges coming out of it:



First Step of Our Construction

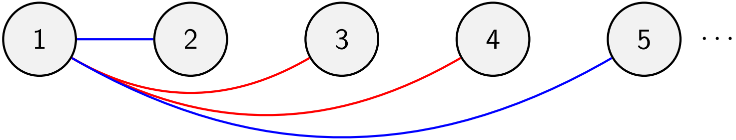
Look at 1 and all of the edges coming out of it:



Either $\exists^\infty$ **R** or $\exists^\infty$ of **B** coming out of 1. We assume **R**.

First Step of Our Construction

Look at 1 and all of the edges coming out of it:

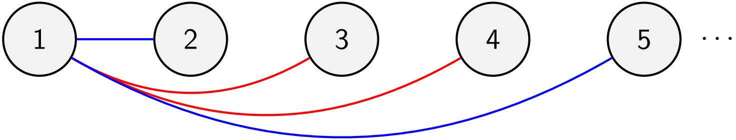


Either $\exists^\infty$ **R** or $\exists^\infty$ of **B** coming out of 1. We assume **R**.

If $\mathrm{COL}(1, y) =$ **R** we say that y **agrees**.

First Step of Our Construction

Look at 1 and all of the edges coming out of it:

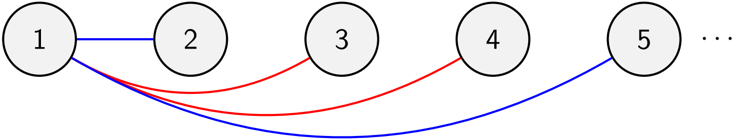


Either $\exists^{\infty}$ **R** or $\exists^{\infty}$ of **B** coming out of 1. We assume **R**.

If $\mathrm{COL}(1, y) =$ **R** we say that y **agrees**.
If $\mathrm{COL}(1, y) \neq$ **R** we say that y **disagrees**.

First Step of Our Construction

Look at 1 and all of the edges coming out of it:



Either $\exists^\infty$ **R** or $\exists^\infty$ of **B** coming out of 1. We assume **R**.

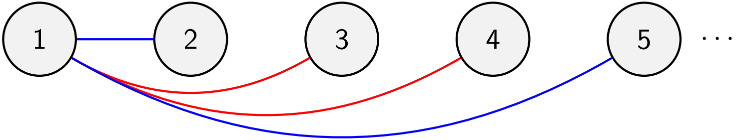
If $\mathrm{COL}(1, y) = $ **R** we say that y **agrees**.

If $\mathrm{COL}(1, y) \neq $ **R** we say that y **disagrees**.

Kill all those who disagree!

First Step of Our Construction

Look at 1 and all of the edges coming out of it:



Either $\exists^{\infty}$ **R** or $\exists^{\infty}$ of **B** coming out of 1. We assume **R**.

If $\mathrm{COL}(1, y) =$ **R** we say that y **agrees**.
If $\mathrm{COL}(1, y) \neq$ **R** we say that y **disagrees**.

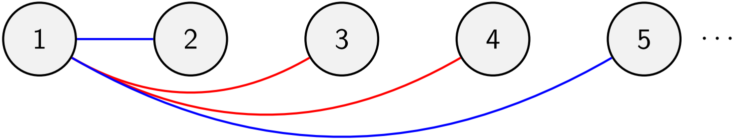
Kill all those who disagree!
We have a picture of this on the next slide.

Node 1 Has the Reds

Look at 1 and all of the edges coming out of it:

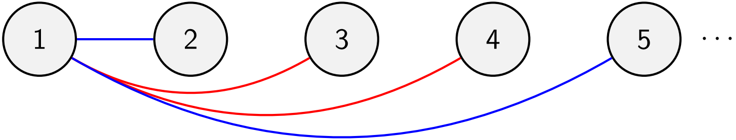
Node 1 Has the Reds

Look at 1 and all of the edges coming out of it:



Node 1 Has the Reds

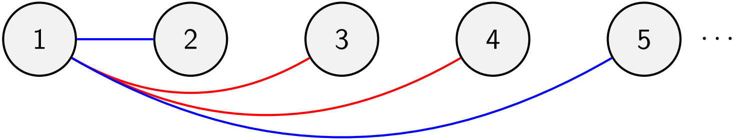
Look at 1 and all of the edges coming out of it:



Assume that 1 has an infinite number of **R** coming out of it.

Node 1 Has the Reds

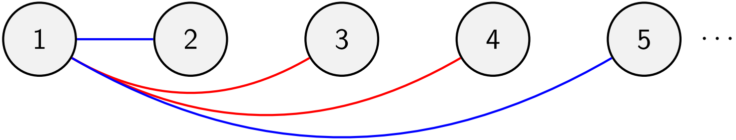
Look at 1 and all of the edges coming out of it:



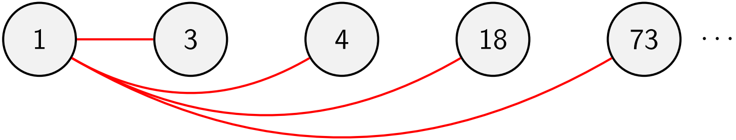
Assume that 1 has an infinite number of **R** coming out of it.
We **kill** 2,5, and anyone else who disagrees!

Node 1 Has the Reds

Look at 1 and all of the edges coming out of it:

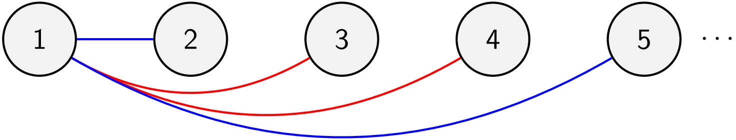


Assume that 1 has an infinite number of **R** coming out of it.
We **kill** 2,5, and anyone else who disagrees!

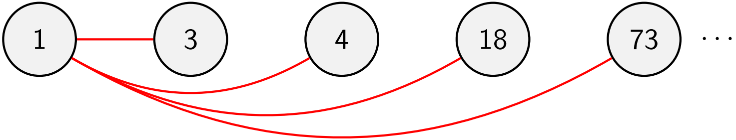


Node 1 Has the Reds

Look at 1 and all of the edges coming out of it:



Assume that 1 has an infinite number of **R** coming out of it.
We **kill** 2,5, and anyone else who disagrees!



We Omit 1 from future pictures but its **Still Alive and Well.**
`https://www.youtube.com/watch?v=8--jVqaU-G8`.

The Next Step

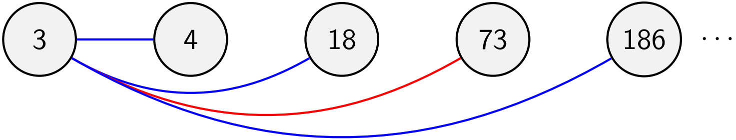
The Next Step

There is a **R** edge from 1 to $3, 4, 18, 73, 186, \ldots$; however, this puts no constraint on the colorings between those nodes.

The Next Step

There is a **R** edge from 1 to $3, 4, 18, 73, 186, \ldots$; however, this puts no constraint on the colorings between those nodes.

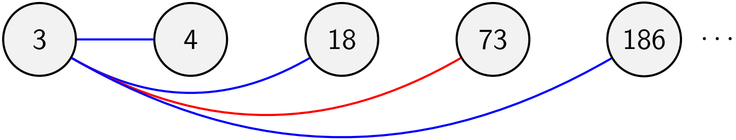
Look at 3 and all of the edges coming out of it.



The Next Step

There is a **R** edge from 1 to $3, 4, 18, 73, 186, \ldots$; however, this puts no constraint on the colorings between those nodes.

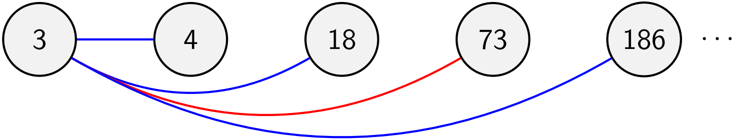
Look at 3 and all of the edges coming out of it.



Either $\exists^\infty$ **R** or $\exists^\infty$ of **B** coming out of 3. We assume **B**.

The Next Step

There is a **R** edge from 1 to $3, 4, 18, 73, 186, \ldots$; however, this puts no constraint on the colorings between those nodes.
Look at 3 and all of the edges coming out of it.



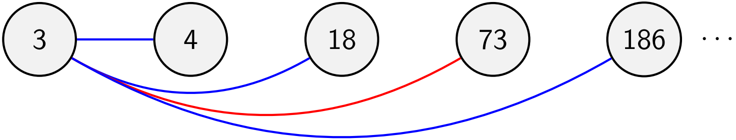
Either $\exists^\infty$ **R** or $\exists^\infty$ of **B** coming out of 3. We assume **B**.

If $\mathrm{COL}(3, y) =$ **B** we say that y **agrees**.

The Next Step

There is a **R** edge from 1 to $3, 4, 18, 73, 186, \ldots$; however, this puts no constraint on the colorings between those nodes.

Look at 3 and all of the edges coming out of it.



Either $\exists^\infty$ **R** or $\exists^\infty$ of **B** coming out of 3. We assume **B**.

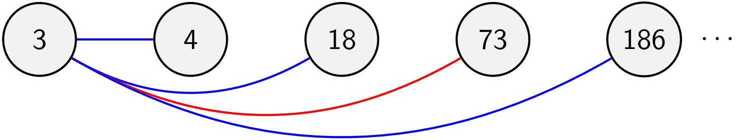
If $\mathrm{COL}(3, y) =$ **B** we say that y **agrees**.

If $\mathrm{COL}(3, y) \neq$ **B** we say that y **disagrees**.

The Next Step

There is a **R** edge from 1 to $3, 4, 18, 73, 186, \ldots$; however, this puts no constraint on the colorings between those nodes.

Look at 3 and all of the edges coming out of it.



Either $\exists^{\infty}$ **R** or $\exists^{\infty}$ of **B** coming out of 3. We assume **B**.

If $\mathrm{COL}(3, y) =$ **B** we say that y **agrees**.

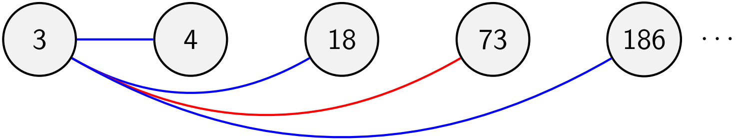
If $\mathrm{COL}(3, y) \neq$ **B** we say that y **disagrees**.

Kill all those who disagree!

The Next Step

There is a **R** edge from 1 to $3, 4, 18, 73, 186, \ldots$; however, this puts no constraint on the colorings between those nodes.

Look at 3 and all of the edges coming out of it.



Either $\exists^\infty$ **R** or $\exists^\infty$ of **B** coming out of 3. We assume **B**.

If $\mathrm{COL}(3, y) =$ **B** we say that y **agrees**.

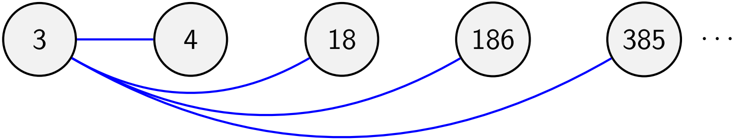
If $\mathrm{COL}(3, y) \neq$ **B** we say that y **disagrees**.

Kill all those who disagree!

We have a picture of this on the next slide.

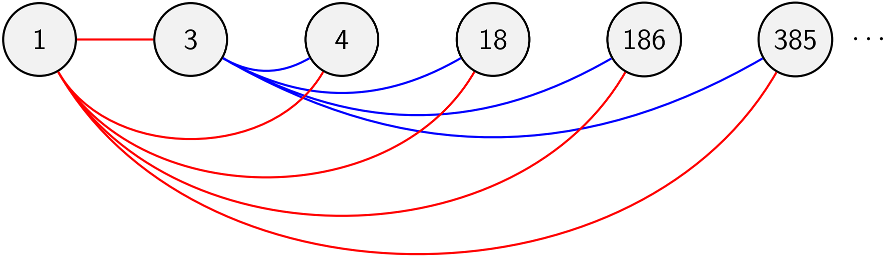
Node 3 Has The Blues

Node 3 Has The Blues

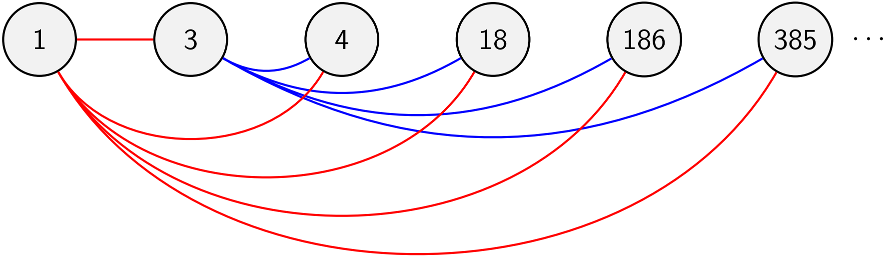


Node 1 and Node 3

Node 1 and Node 3

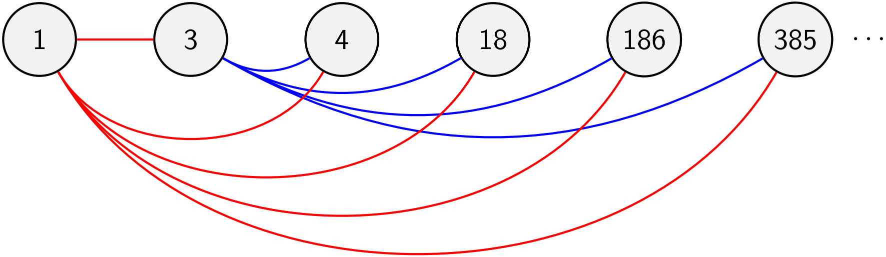


Node 1 and Node 3



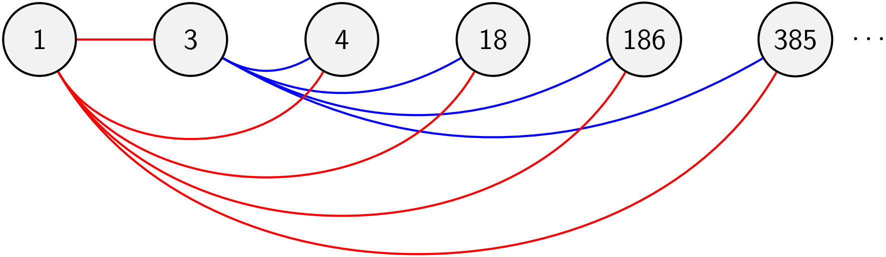
We could keep doing this with node 4, but messy!

Node 1 and Node 3



We could keep doing this with node 4, but messy!
Note that at this point nodes 1 and 3 cannot be killed.

Node 1 and Node 3



We could keep doing this with node 4, but messy!
Note that at this point nodes 1 and 3 cannot be killed.
We formalize the real construction on the next slides.

Given $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ We Form COL'

We said earlier

Either $\exists^{\infty}$ R or $\exists^{\infty}$ of B coming out of 1

When we formalize this, we will **color** node 1 with that color.

Given $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ We Form COL'

We said earlier

Either $\exists^\infty$ R or $\exists^\infty$ of B coming out of 1

When we formalize this, we will **color** node 1 with that color.

We will then kill all nodes who disagree, but, and this is key

Given $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ We Form COL'

We said earlier

Either $\exists^\infty$ R or $\exists^\infty$ of B coming out of 1

When we formalize this, we will **color** node 1 with that color.

We will then kill all nodes who disagree, but, and this is key

Still have an Infinite Number of Nodes In Play.

Formal Construction

Formal Construction

$H_1 = \mathbb{N}$

Formal Construction

$H_1 = \mathbb{N}$
$x_1 = 1$

Formal Construction

$H_1 = \mathbb{N}$
$x_1 = 1$
$c_1 =$ **R** if $|\{y \in H_1 \colon \mathrm{COL}(x_1, y) = \textbf{R}\}| = \infty$, **B** otherwise.

Formal Construction

$H_1 = \mathbb{N}$
$x_1 = 1$
$c_1 = \mathbf{R}$ if $|\{y \in H_1 : \mathrm{COL}(x_1, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

$H_2 = \{x_1\} \cup \{y \in H_1 : \mathrm{COL}(x_1, y) = c_1\}$

Formal Construction

$H_1 = \mathbb{N}$
$x_1 = 1$
$c_1 =$ **R** if $|\{y \in H_1 : \mathrm{COL}(x_1, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

$H_2 = \{x_1\} \cup \{y \in H_1 : \mathrm{COL}(x_1, y) = c_1\}$
$x_2 =$ the least element of $H_2 - \{x_1\}$.

Formal Construction

$H_1 = \mathbb{N}$
$x_1 = 1$
$c_1 =$ **R** if $|\{y \in H_1 : \mathrm{COL}(x_1, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

$H_2 = \{x_1\} \cup \{y \in H_1 : \mathrm{COL}(x_1, y) = c_1\}$
$x_2 =$ the least element of $H_2 - \{x_1\}$.
$c_2 =$ **R** if $|\{y \in H_2 : \mathrm{COL}(x_2, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

Formal Construction

$H_1 = \mathbb{N}$
$x_1 = 1$
$c_1 =$ **R** if $|\{y \in H_1 \colon \mathrm{COL}(x_1, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

$H_2 = \{x_1\} \cup \{y \in H_1 \colon \mathrm{COL}(x_1, y) = c_1\}$
$x_2 =$ the least element of $H_2 - \{x_1\}$.
$c_2 =$ **R** if $|\{y \in H_2 \colon \mathrm{COL}(x_2, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

Assume H_s, x_s, c_s are defined.

Formal Construction

$H_1 = \mathbb{N}$
$x_1 = 1$
$c_1 =$ **R** if $|\{y \in H_1 \colon \mathrm{COL}(x_1, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

$H_2 = \{x_1\} \cup \{y \in H_1 \colon \mathrm{COL}(x_1, y) = c_1\}$
$x_2 =$ the least element of $H_2 - \{x_1\}$.
$c_2 =$ **R** if $|\{y \in H_2 \colon \mathrm{COL}(x_2, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

Assume H_s, x_s, c_s are defined.
$H_{s+1} = \{x_s\} \cup \{y \in H_s \colon \mathrm{COL}(x_s, y) = c_s\}$

Formal Construction

$H_1 = \mathbb{N}$
$x_1 = 1$
$c_1 = \mathbf{R}$ if $|\{y \in H_1 : \mathrm{COL}(x_1, y) = \mathbf{R}\}| = \infty$, $\mathbf{B}$ otherwise.

$H_2 = \{x_1\} \cup \{y \in H_1 : \mathrm{COL}(x_1, y) = c_1\}$
$x_2 =$ the least element of $H_2 - \{x_1\}$.
$c_2 = \mathbf{R}$ if $|\{y \in H_2 : \mathrm{COL}(x_2, y) = \mathbf{R}\}| = \infty$, $\mathbf{B}$ otherwise.

Assume H_s, x_s, c_s are defined.
$H_{s+1} = \{x_s\} \cup \{y \in H_s : \mathrm{COL}(x_s, y) = c_s\}$
$x_{s+1} =$ the least element of $H_{s+1} - \{x_1, \ldots, x_s\}$.

Formal Construction

$H_1 = \mathbb{N}$
$x_1 = 1$
$c_1 =$ **R** if $|\{y \in H_1 : \mathrm{COL}(x_1, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

$H_2 = \{x_1\} \cup \{y \in H_1 : \mathrm{COL}(x_1, y) = c_1\}$
$x_2 =$ the least element of $H_2 - \{x_1\}$.
$c_2 =$ **R** if $|\{y \in H_2 : \mathrm{COL}(x_2, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

Assume H_s, x_s, c_s are defined.
$H_{s+1} = \{x_s\} \cup \{y \in H_s : \mathrm{COL}(x_s, y) = c_s\}$
$x_{s+1} =$ the least element of $H_{s+1} - \{x_1, \ldots, x_s\}$.
$c_{s+1} =$ **R** if $|\{y \in H_{s+1} : \mathrm{COL}(x_{s+1}, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

Formal Construction

$H_1 = \mathbb{N}$
$x_1 = 1$
$c_1 =$ **R** if $|\{y \in H_1 \colon \mathrm{COL}(x_1, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

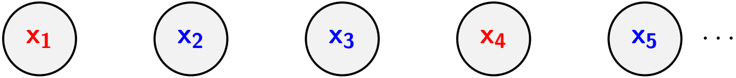
$H_2 = \{x_1\} \cup \{y \in H_1 \colon \mathrm{COL}(x_1, y) = c_1\}$
$x_2 =$ the least element of $H_2 - \{x_1\}$.
$c_2 =$ **R** if $|\{y \in H_2 \colon \mathrm{COL}(x_2, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

Assume H_s, x_s, c_s are defined.
$H_{s+1} = \{x_s\} \cup \{y \in H_s \colon \mathrm{COL}(x_s, y) = c_s\}$
$x_{s+1} =$ the least element of $H_{s+1} - \{x_1, \ldots, x_s\}$.
$c_{s+1} =$ **R** if $|\{y \in H_{s+1} \colon \mathrm{COL}(x_{s+1}, y) = \mathbf{R}\}| = \infty$, **B** otherwise.

$$X = \{x_1, x_2, \ldots\}$$

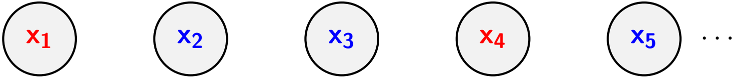
The Coloring of the Nodes

The Coloring of the Nodes



All of the edges from x_1 to the left are **R**.

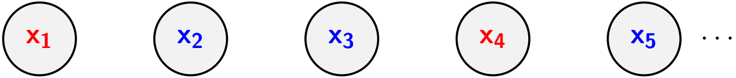
The Coloring of the Nodes



All of the edges from x_1 to the left are **R**.

All of the edges from x_2 to the left are **B**.

The Coloring of the Nodes

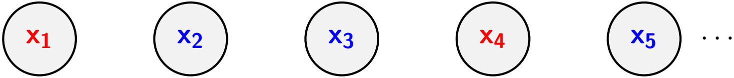


All of the edges from x_1 to the left are **R**.

All of the edges from x_2 to the left are **B**.

All of the edges from x_3 to the left are **B**.

The Coloring of the Nodes



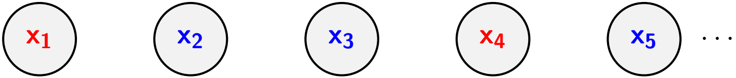
All of the edges from x_1 to the left are **R**.

All of the edges from x_2 to the left are **B**.

All of the edges from x_3 to the left are **B**.

All of the edges from x_4 to the left are **R**.

The Coloring of the Nodes



All of the edges from x_1 to the left are **R**.

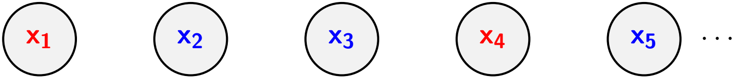
All of the edges from x_2 to the left are **B**.

All of the edges from x_3 to the left are **B**.

All of the edges from x_4 to the left are **R**.

All of the edges from x_5 to the left are **B**.

The Coloring of the Nodes



All of the edges from x_1 to the left are **R**.

All of the edges from x_2 to the left are **B**.

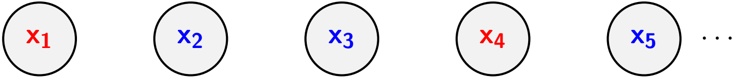
All of the edges from x_3 to the left are **B**.

All of the edges from x_4 to the left are **R**.

All of the edges from x_5 to the left are **B**.

All of the edges from x_s to the left are c_s.

The Coloring of the Nodes



All of the edges from x_1 to the left are **R**.

All of the edges from x_2 to the left are **B**.

All of the edges from x_3 to the left are **B**.

All of the edges from x_4 to the left are **R**.

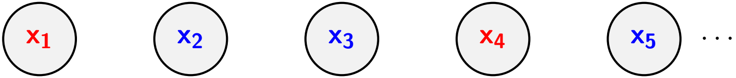
All of the edges from x_5 to the left are **B**.

All of the edges from $\mathbf{x_s}$ to the left are $\mathbf{c_s}$.

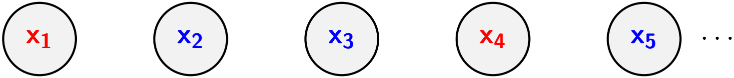
What do you think our next step is?

Some Color Appears Infinitely Often

Some Color Appears Infinitely Often

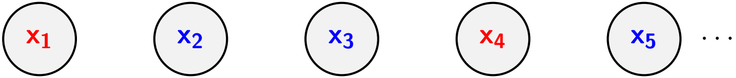


Some Color Appears Infinitely Often



All of the edges from $\mathbf{x_s}$ to the left are $\mathbf{c_s}$.

Some Color Appears Infinitely Often

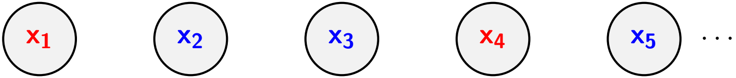


All of the edges from $\mathbf{x_s}$ to the left are $\mathbf{c_s}$.

Some color appears infinitely often, say **R**.

$$H = \{y \in X : \mathrm{COL}(y) = \mathbf{R}\}$$

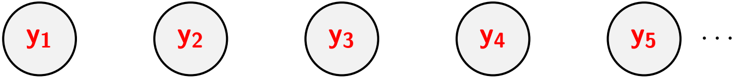
Some Color Appears Infinitely Often



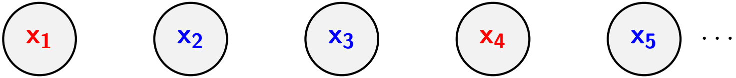
All of the edges from $\mathbf{x_s}$ to the left are $\mathbf{c_s}$.

Some color appears infinitely often, say **R**.

$$H = \{y \in X : \mathrm{COL}(y) = \mathbf{R}\}$$



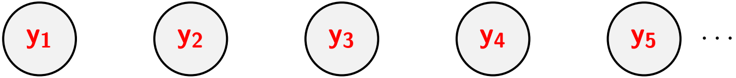
Some Color Appears Infinitely Often



All of the edges from $\mathbf{x_s}$ to the left are $\mathbf{c_s}$.

Some color appears infinitely often, say **R**.

$$H = \{y \in X : \mathrm{COL}(y) = \mathbf{R}\}$$



All of the edges from $\mathbf{y_s}$ to the left are **R**.

Some Color Appears Infinitely Often

x_1 x_2 x_3 x_4 x_5 $\cdots$

All of the edges from $\mathbf{x_s}$ to the left are $\mathbf{c_s}$.

Some color appears infinitely often, say **R**.

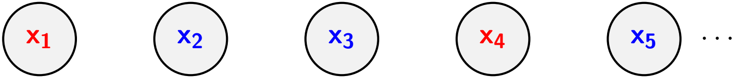
$$H = \{y \in X : \mathrm{COL}(y) = \mathbf{R}\}$$

y_1 y_2 y_3 y_4 y_5 $\cdots$

All of the edges from $\mathbf{y_s}$ to the left are **R**.

H is clearly a homog set!

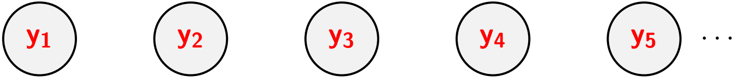
Some Color Appears Infinitely Often



All of the edges from $\mathbf{x_s}$ to the left are $\mathbf{c_s}$.

Some color appears infinitely often, say **R**.

$$H = \{y \in X : \mathrm{COL}(y) = \mathbf{R}\}$$



All of the edges from $\mathbf{y_s}$ to the left are **R**.

H is clearly a homog set!

DONE!

Variants Of
The
Infinite Ramsey Theorem

What if There Are c Colors?

We proved

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \rightarrow [2]$ there exists an infinite homog set.

What if There Are c Colors?

We proved
Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ there exists an infinite homog set.

What if we use c colors? Is the following true?

What if There Are c Colors?

We proved
Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ there exists an infinite homog set.

What if we use c colors? Is the following true?

Thm For all $c \in \mathbb{N}$, for all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [c]$ there exists an infinite homog set.

What if There Are c Colors?

We proved

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \rightarrow [2]$ there exists an infinite homog set.

What if we use c colors? Is the following true?

Thm For all $c \in \mathbb{N}$, for all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \rightarrow [c]$ there exists an infinite homog set.

This is easy to prove using the same technique we used for the $c = 2$ case.

What About Hypergraphs?

What About Hypergraphs?

Def

What About Hypergraphs?

Def

1) A **1-hypergraph** is (V, E) where $E \subseteq \binom{V}{1}$.

What About Hypergraphs?

Def

1) A **1-hypergraph** is (V, E) where $E \subseteq \binom{V}{1}$.
2) A **2-hypergraph** is (V, E) where $E \subseteq \binom{V}{2}$. This is just a graph.

What About Hypergraphs?

Def

1) A **1-hypergraph** is (V, E) where $E \subseteq \binom{V}{1}$.
2) A **2-hypergraph** is (V, E) where $E \subseteq \binom{V}{2}$. This is just a graph.
3) A **3-hypergraph** is (V, E) where $E \subseteq \binom{V}{3}$.

What About Hypergraphs?

Def
1) A **1-hypergraph** is (V, E) where $E \subseteq \binom{V}{1}$.
2) A **2-hypergraph** is (V, E) where $E \subseteq \binom{V}{2}$. This is just a graph.
3) A **3-hypergraph** is (V, E) where $E \subseteq \binom{V}{3}$.
a) An **a-hypergraph** is (V, E) where $E \subseteq \binom{V}{a}$.

What About Hypergraphs?

Def
1) A **1-hypergraph** is (V, E) where $E \subseteq \binom{V}{1}$.
2) A **2-hypergraph** is (V, E) where $E \subseteq \binom{V}{2}$. This is just a graph.
3) A **3-hypergraph** is (V, E) where $E \subseteq \binom{V}{3}$.
a) An **a-hypergraph** is (V, E) where $E \subseteq \binom{V}{a}$.

We proved the Infinite Ramsey Thm for 2-hypergraphs.

What About Hypergraphs?

Def
1) A **1-hypergraph** is (V, E) where $E \subseteq \binom{V}{1}$.
2) A **2-hypergraph** is (V, E) where $E \subseteq \binom{V}{2}$. This is just a graph.
3) A **3-hypergraph** is (V, E) where $E \subseteq \binom{V}{3}$.
a) An **a-hypergraph** is (V, E) where $E \subseteq \binom{V}{a}$.

We proved the Infinite Ramsey Thm for 2-hypergraphs.

We will define homog sets for colorings of the edges of an a-hypergraph and prove **Infinite Ramsey Thm for a-hypergraphs.**

What About Hypergraphs?

Def
1) A **1-hypergraph** is (V, E) where $E \subseteq \binom{V}{1}$.
2) A **2-hypergraph** is (V, E) where $E \subseteq \binom{V}{2}$. This is just a graph.
3) A **3-hypergraph** is (V, E) where $E \subseteq \binom{V}{3}$.
a) An **a-hypergraph** is (V, E) where $E \subseteq \binom{V}{a}$.

We proved the Infinite Ramsey Thm for 2-hypergraphs.

We will define homog sets for colorings of the edges of an a-hypergraph and prove **Infinite Ramsey Thm for a-hypergraphs.**
In a later lecture.

What About Finite Ramsey?

We showed **The Infinite Ramsey Thm For 2-Hypergraphs**

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \rightarrow [2]$ there exists an infinite homog set.

What About Finite Ramsey?

We showed **The Infinite Ramsey Thm For 2-Hypergraphs**

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ there exists an infinite homog set.

In a later lecture we will use

What About Finite Ramsey?

We showed **The Infinite Ramsey Thm For 2-Hypergraphs**

Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ there exists an infinite homog set.

In a later lecture we will use

The Infinite Ramsey Thm For 2-Hypergraphs to prove

What About Finite Ramsey?

We showed **The Infinite Ramsey Thm For 2-Hypergraphs**
Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \rightarrow [2]$ there exists an infinite homog set.

In a later lecture we will use
The Infinite Ramsey Thm For 2-Hypergraphs to prove
The Finite Ramsey Thm for 2-Hypergraphs:

What About Finite Ramsey?

We showed **The Infinite Ramsey Thm For 2-Hypergraphs**
Thm For all $\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ there exists an infinite homog set.

In a later lecture we will use
The Infinite Ramsey Thm For 2-Hypergraphs to prove
The Finite Ramsey Thm for 2-Hypergraphs:
Thm For all k there exists $n = R(k)$ such that for all
$\mathrm{COL}\colon \binom{\mathbb{N}}{2} \to [2]$ there exists a homog set of size k.