Infinite Ramsey Theorem For 3-Hypergraph

Exposition by William Gasarch

December 10, 2024

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We do some an example of the first few steps of the construction.

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What to make of this? Discuss.

Construct a Coloring of $\binom{\mathbb{N}}{2}$

We are given $\mathrm{COL}\colon \binom{\mathbb{N}}{3} \to [2]$.

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For all $y,z \in \mathbb{N}-\{1\}$, $\mathrm{COL}(1,y,z) = c_1$.

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If $y \in H_1$ we say that y **agrees**.
If $y \notin H_2$ we say that y **disagrees**.

Kill all those who disagree!

Construction of $x_1, H_1, c_1, x_2, H_2, c_2$

We now have

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x_2 is the least element of H_1.

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Next Slide is General Case.

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c_{s+1} is the color of H_{s+1}.

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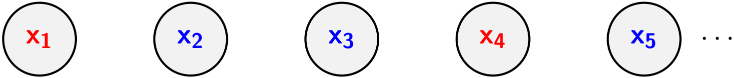

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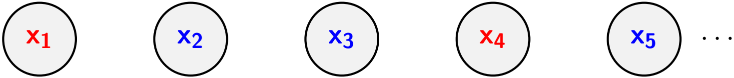

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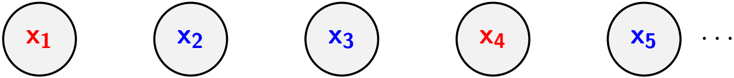

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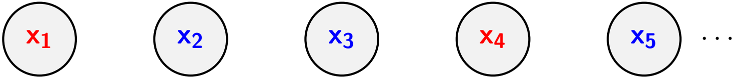

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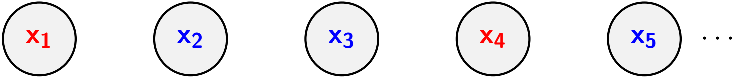

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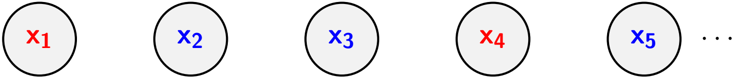

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What do you think our next step is?

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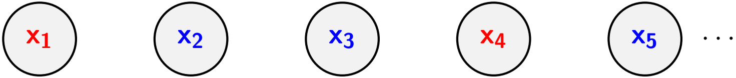

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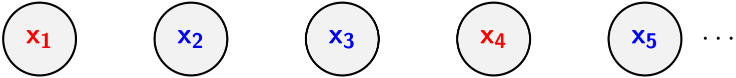

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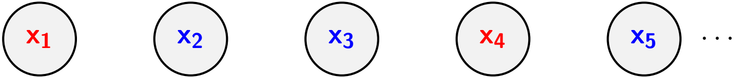

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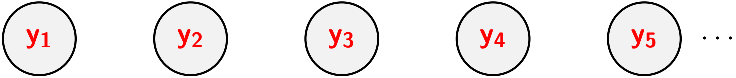

For all $i < j < k$, $\mathrm{COL}(x_i, x_j, x_k) = \mathbf{R}$. (More generally c.)

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$a = 1$: $\forall$ 2-colorings of $\mathbb{N}$ some color appears ∞. The set of $x \in \mathbb{N}$ of that color is an infinite homog set.

$a = 2$: ∞ Ramsey Thm for Graphs. Our proof used $a = 1$ case.

$a = 3$: These Slides!. We will use $a = 1$ and $a = 2$ cases.

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$a \ge 4$: Might be a HW. Should be easy for you now.

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That proof easily extends to $R_a(k)$.