Exposition by William Gasarch-U of MD

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Writing a number as a sum of powers of 2.

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We can even write the exponents that are not already powers of 2 as sums of powers of 2.

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This is called Hereditary Base n Notation

$$1000 = 2^{2^{2^1+2^0}+2^0} + 2^{2^{2^1+2^0}} + 2^{2^2+2^1+2^0} + 2^{2^1+2^0}$$

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$$1000 = 2^{2^{2^{1}+2^{0}}+2^{0}} + 2^{2^{2^{1}+2^{0}}} + 2^{2^{2}+2^{1}+2^{0}} + 2^{2^{1}+2^{0}}$$

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Repeat the process:

Replace 3 by 4, and subtract 1, Replace 4 by 5, and subtract 1, ....

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- Goto infinity (and if so how fast- perhaps Ack-like?)
- Eventually stabilizes (e.g., goes to 18 and then stops there)
- Cycles- goes UP then DOWN then UP then DOWN ....

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The seq goes to 0.



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The number of steps for n to go to 0 is roughly ACK(n, n).



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The seq goes to 0. The number of steps for n to goto 0 is roughly ACK(n, n). Really? Really!

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Increase the base and subtract 1. Assume BWOC that the seq goes on forever.

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Repeat this to get:  $(984)_{12}$ ,  $(983)_{13}$ ,  $(982)_{14}$ ,  $(981)_{15}$ ,  $(980)_{16}$ .

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Note that the right most digit is 0. That will happen  $\infty$  often.

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 $\rightarrow (96(23))_{34} \rightarrow (960)_{57} \rightarrow (95(47))_{58}$ 

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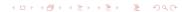
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$$(95(47))_{48} \to \cdots \to (900)_y$$

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#### Weak Goodstein: Second Position

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Now its a 2-digit number and use induction.

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1. If original number is 1-digit long then it will clearly go to 0.

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**Goodstein's Thm** The strong Goodstein seq always goes to 0. Do you find his theorem to be natural? This is not a VOTE since it's a matter of opinion and **natural** is not well defined.

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Goodstein's Thm The strong Goodstein seq always goes to 0.

Do you find his theorem to be natural? This is not a VOTE since it's a matter of opinion and **natural** is not well defined.

Next Slide will indicate why am asking this.

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- 2. Finitary versions of Kruskal's Tree Theorem.
- 3. Harvey Friedman has done much research on this. Here is one of his theorems:

https:

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files/2014/01/FliniteSeqInc062214a-v9w7q4.pdf