Finite Ramsey Theorem For Graphs

Exposition by William Gasarch

December 8, 2024

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(a) every pair of elements of H knows each other, or
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Examples of The First Few Steps of The Construction

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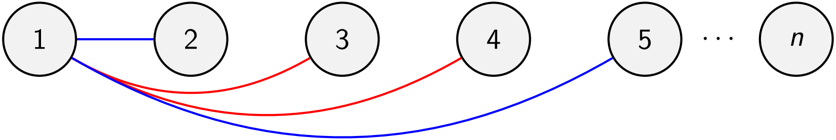

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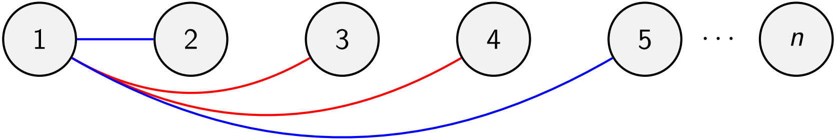

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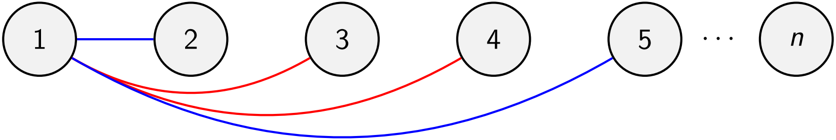

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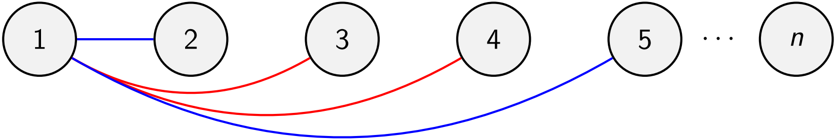

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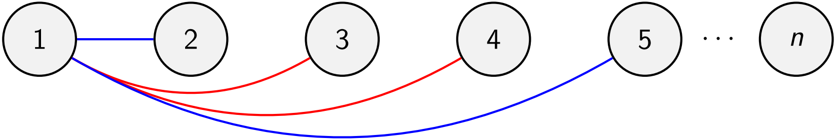

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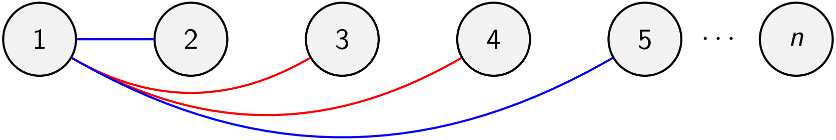

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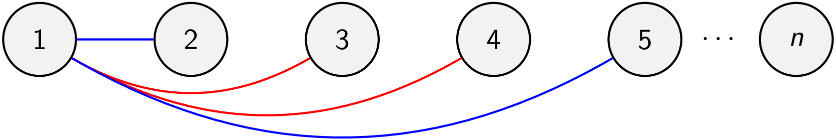

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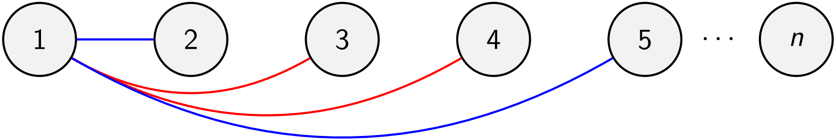

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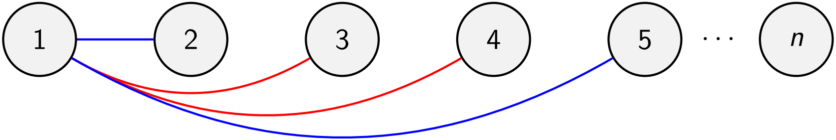

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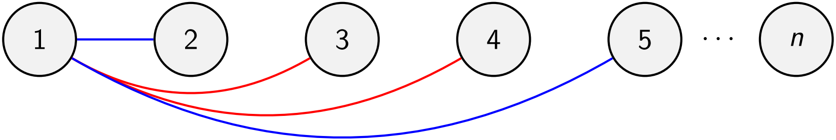

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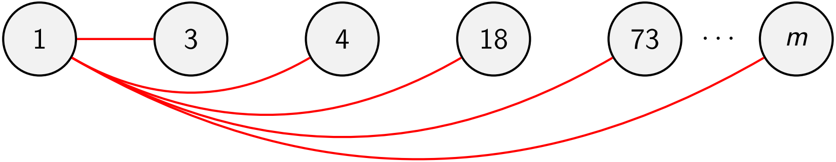

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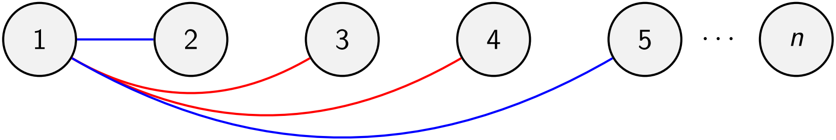

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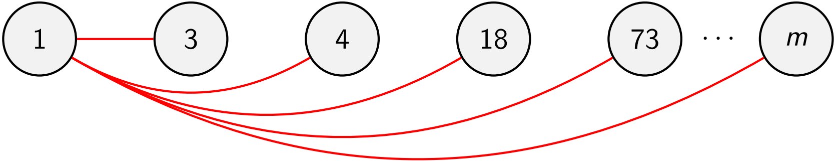

We Omit 1 from future pictures but its **Still Alive and Well.**
`https://www.youtube.com/watch?v=8--jVqaU-G8`.

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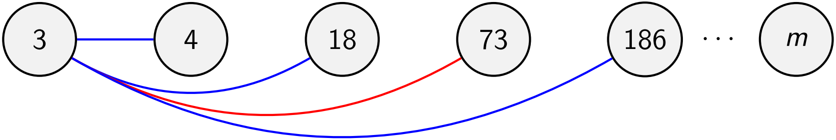

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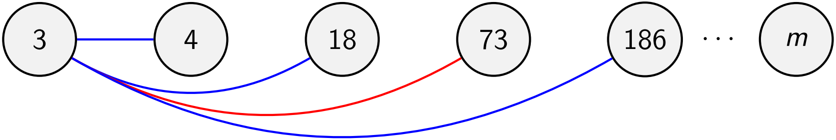

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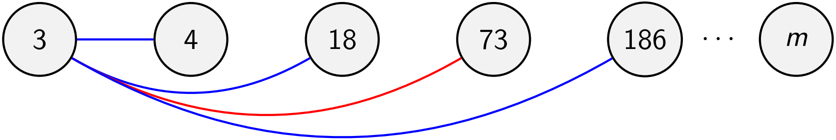

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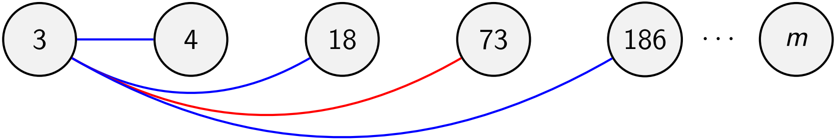

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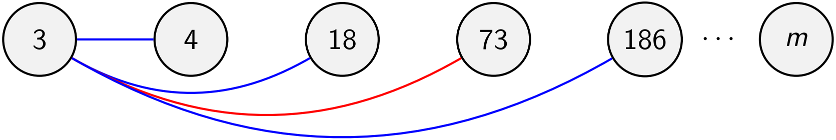

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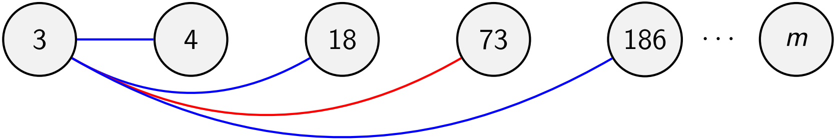

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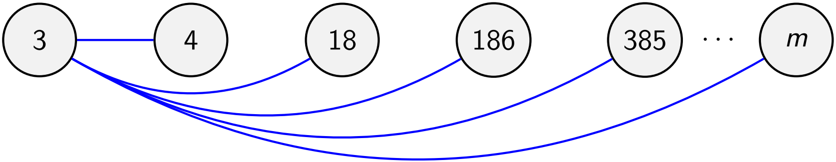

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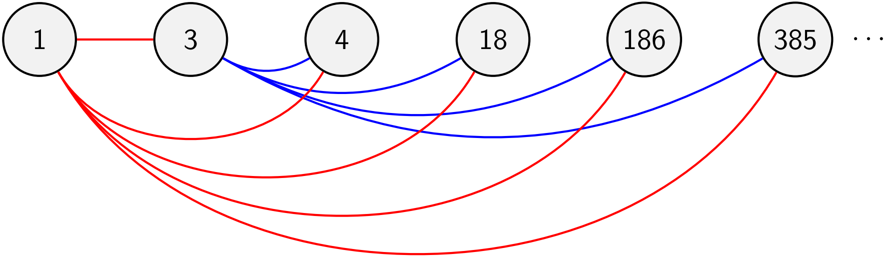

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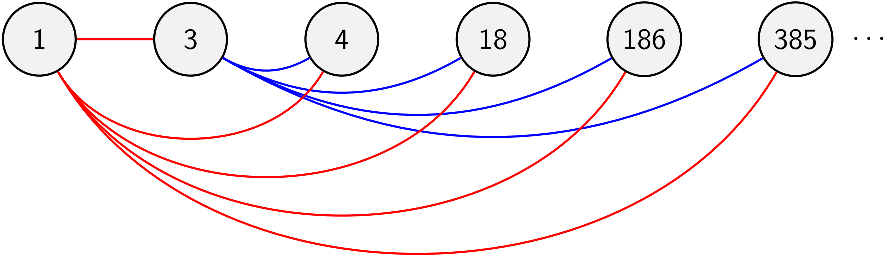

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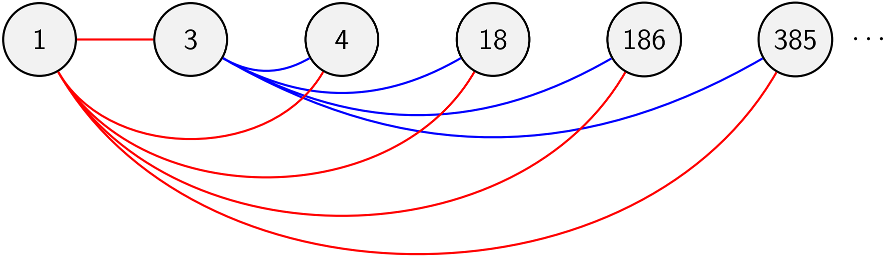

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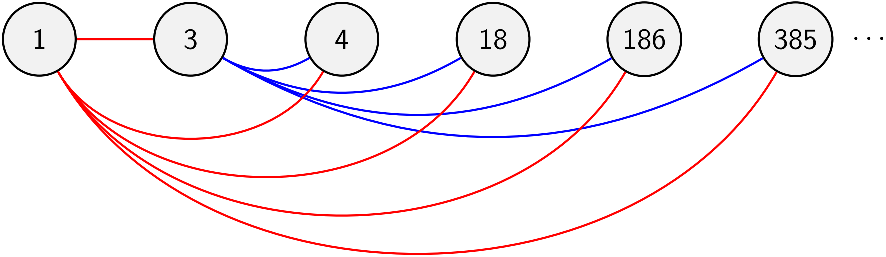

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After s stages still have $n/2^s$ Nodes In Play.

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But the ... is NOT infinite. Where to stop? See next slide

The Size of X

We will see later than we want $|X| \geq 2^{2k-1}$.

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It suffices to take $n \geq 2^{2k-1}$.

The Coloring of the Nodes

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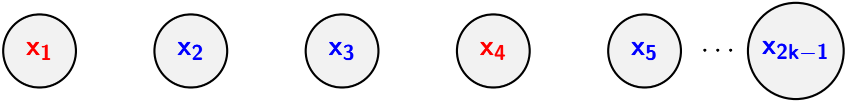

All of the edges from x_1 to the left are **R**.

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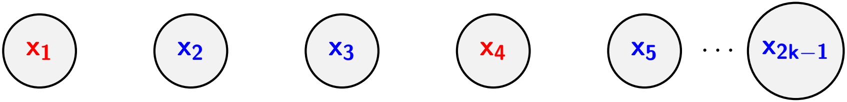

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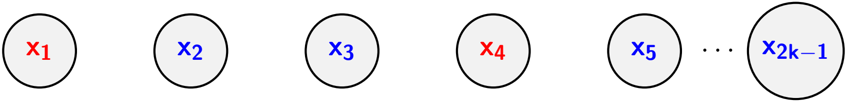

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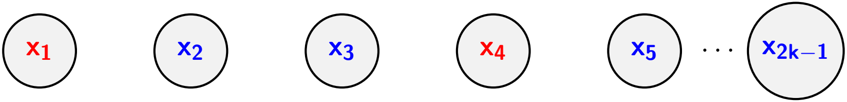

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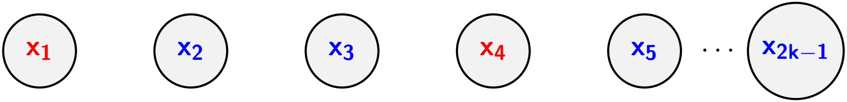

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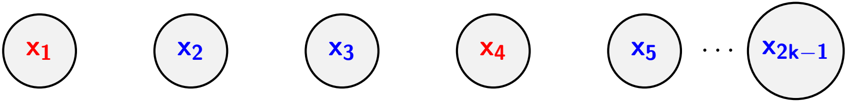

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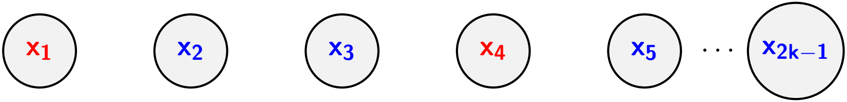

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What do you think our next step is?

Some Color Appears k Times

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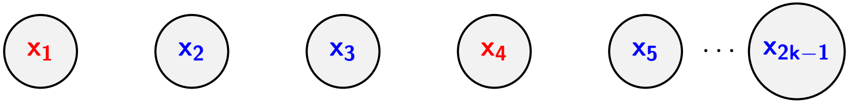

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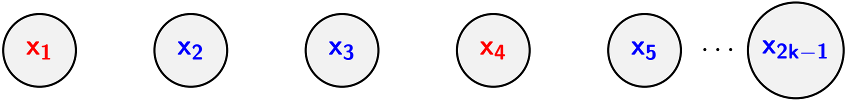

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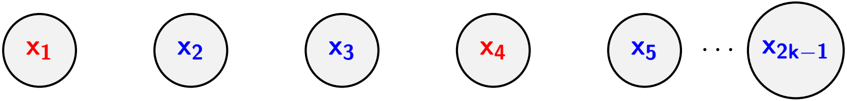

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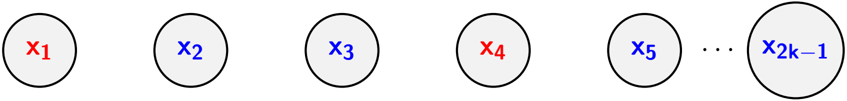

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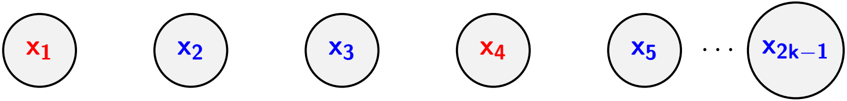

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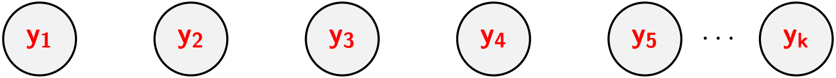

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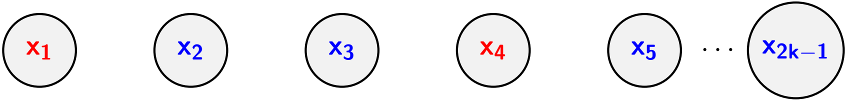

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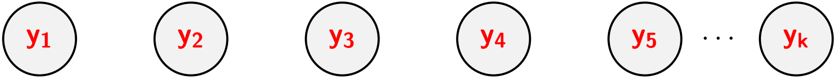

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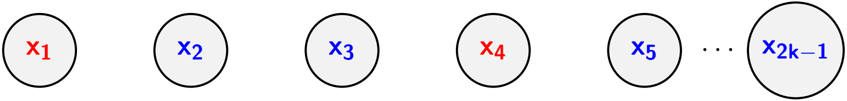

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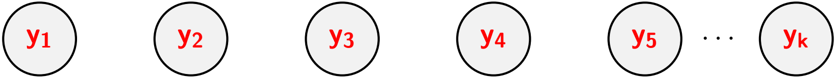

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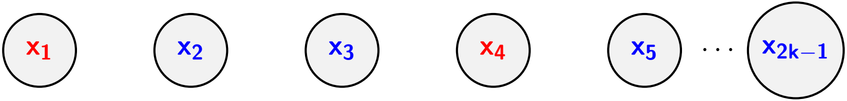

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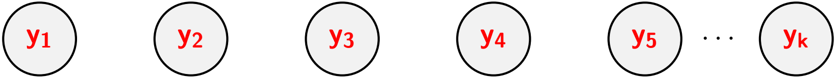

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DONE!

Variants Of
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This is easy to prove using the same technique we used for the $c = 2$ case.

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