

# Project On Shift, Affine, Quad Cipher

# Shift Cipher Project

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To avoid confusing 21 with 2 1 I used dashes above. You can use a different mechanism.

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5) Input a text of English and a shift  $s \in \{0, \dots, 25\}$ . Output the result of shifting the text by  $s$ . This will just be combining the programs above.

# Cracking the Shift Cipher

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For the last step we need a **program** that can tell if a text **looks like English**

# Freq Vectors

Let  $T$  be a long text. Length  $N$ . May or may not be coded.

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The **Freq Vector of  $T$**  is

$$\vec{f}_T = \left( \frac{N_a}{N}, \frac{N_b}{N}, \dots, \frac{N_z}{N} \right)$$

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- ▶  $\sum_{i=1}^{25} f_{E,i} f_{T,i}$  If large then IS-ENGLISH. Need to find out how large. This one is actually used.



# How to Find Parameters

Let  $f(T) = \sum_{i=0}^{25} |f_{E,i} - f_{T,i}|$

We want numbers  $\alpha, \beta$  such that

- ▶ If  $T$  is English then  $f(T) \geq \alpha$ .
- ▶ If  $T$  is English SHIFTED by a non-zero number then  $f(T) \leq \beta$ .
- ▶  $\alpha - \beta$  is large.

How to find  $\alpha, \beta$ ?

Next slide.

# Program to Find $\alpha, \beta$ , Method 1

Let  $T$  be a large English Text.

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SO, are these the  $\alpha, \beta$  you should use? You should do this for a few more texts and let  $\alpha$  be the max of the  $\alpha$ 's, and  $\beta$  be the min of the  $\beta$ 's.

## Program to Find $\alpha, \beta$ , Method 2

Do the same with

$$\sum_{i=0}^{25} (f_{E,i} - f_{T,i})^2$$

## Program to Find $\alpha, \beta$ , Method 3

Do the same with

$$\sum_{i=1}^{25} f_{E,i} f_{T,i}$$

BUT note here we want minimize  $\alpha$  and maximize  $\beta$ .



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You can try this without removing punctuation.

# Affine Cipher Project

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For each program you wrote for the SHIFT cipher, write a similar one for AFFINE.

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