BILL, RECORD LECTURE!!!!

BILL RECORD LECTURE!!!

Nondeterministic Finite Automata (NFA)

An Interesting Example of a DFA

With neighbor find DFA's for the following. Note numb. states.
$\Sigma^* a$
$\Sigma^* a \Sigma$
$\Sigma^* a \Sigma^2$

$\Sigma^* a \Sigma^2$

https://www.cs.umd.edu/users/gasarch/COURSES/452/S21/notes/dfa3.JPG

$\Sigma^* a \Sigma^2$

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The number of states is 8.

$\Sigma^* a \Sigma^2$

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More generally:
$\Sigma^* a \Sigma^i$ can be done with 2^{i+1} states.

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Prove for $\Sigma^* a \Sigma^3$, with a table.

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Might be on $2^{\{\text{HW, MIDTERM, FINAL}\}}$.

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We may prove this later.

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We now use NFA's informally.

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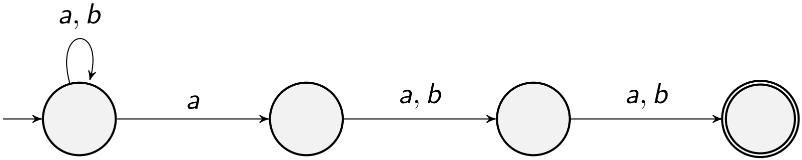
1. From state q, and symbol σ may be ≥ 2 states to go to.
2. From a state q and no symbols there may be ≥ 1 states to go to. (We use e for **empty string**.)
3. An NFA accepts a string if there is **some** way to process the string and get to a final state.

NFA for $\Sigma^* a \Sigma^2$



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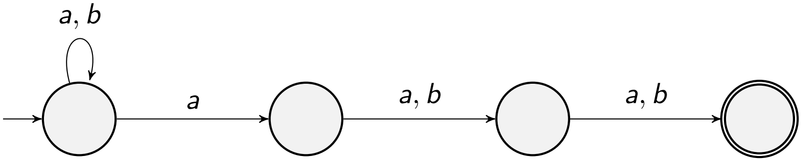

DFA had 8 states. NFA has 4 states.

NFA for $\Sigma^* a \Sigma^3$

Recall that DFA for $\Sigma^* a \Sigma^3$ used 16 states.

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Recall that DFA for $\Sigma^* a \Sigma^3$ used 16 states.

Draw an NFA for $\Sigma^* a \Sigma^3$.

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Make a conjecture for number of states for NFA for $\Sigma^* a \Sigma^n$.

Upshot Seems like NFA uses far fewer state than DFA for $\Sigma^* a \Sigma^n$.

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YES - next slide.

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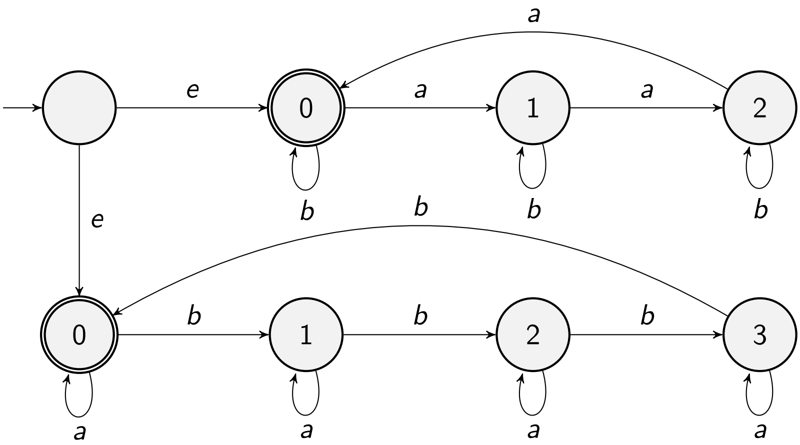

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NO. Proof similar to that for DFA. May do both later.

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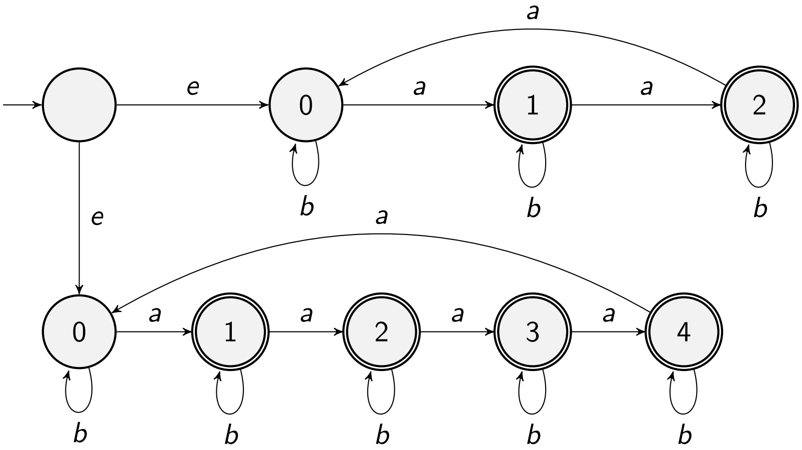

$\{a^n : n \not\equiv 0 \pmod{15}\}$

Prove that the NFA in the last slide works.
Need

$$(n \not\equiv 0 \pmod 3 \vee n \not\equiv 0 \pmod 5) \implies n \not\equiv 0 \pmod{15}$$

Take the contrapositive

$$n \equiv 0 \pmod{15} \implies (n \equiv 0 \pmod 3 \wedge n \equiv 0 \pmod 5)$$

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NO. Proof similar to that for DFA. May do both later, after we define NFA rigorously.

NFA's Intuitively

1. An NFA is a DFA that can guess.
2. NFAs do not really exist.
3. Good for $\cup$ since can guess which one.
4. An NFA accepts iff SOME guess accepts.
5. NFA's are **useful** as intermediary devices.

NFA Formally

Def An **NFA** is a tuple $(Q, \Sigma, \Delta, s, F)$ where:

1. Q is a finite set of **states**.
2. Σ is a finite **alphabet**.
3. $\Delta : Q \times (\Sigma \cup \{e\}) \to 2^Q$ is the *transition function*.
4. $s \in Q$ is the **start state**.
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Def If M is an NFA then $L(M) = \{x : M(x) \text{ accepts } \}$.

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- **Mathematical:** Create tree with branches whenever there is a choice. Accept if any leaf accepts.
- **Magic:** Guess at each nondeterministic step which way to go. Machine always makes right guess if there is one.

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We are nowhere near done. Next slide.

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