BILL, RECORD LECTURE!!!!

BILL RECORD LECTURE!!!

Tricks for Divisibility and DFA's

What I Learned in Junior High School

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.
Let $x = x_n \cdots x_0$.

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.
Let $x = x_n \cdots x_0$.

- 2:

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv 0 \pmod{2}$ iff $x_0 \equiv 0 \pmod{2}$.

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv 0 \pmod{2}$ iff $x_0 \equiv 0 \pmod{2}$.
- 3:

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv 0 \pmod{2}$ iff $x_0 \equiv 0 \pmod{2}$.
- 3: $x \equiv 0 \pmod{3}$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod{3}$.

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv 0 \pmod{2}$ iff $x_0 \equiv 0 \pmod{2}$.
- 3: $x \equiv 0 \pmod{3}$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod{3}$.
- 5:

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.

Let $x = x_n \cdots x_0$.

- 2: $x \equiv 0 \pmod 2$ iff $x_0 \equiv 0 \pmod 2$.
- 3: $x \equiv 0 \pmod 3$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod 3$.
- 5: $x \equiv 0 \pmod 5$ iff $x_0 \equiv 0 \pmod 5$.

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv 0 \pmod 2$ iff $x_0 \equiv 0 \pmod 2$.
- 3: $x \equiv 0 \pmod 3$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod 3$.
- 5: $x \equiv 0 \pmod 5$ iff $x_0 \equiv 0 \pmod 5$.
- 9:

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv 0 \pmod 2$ iff $x_0 \equiv 0 \pmod 2$.
- 3: $x \equiv 0 \pmod 3$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod 3$.
- 5: $x \equiv 0 \pmod 5$ iff $x_0 \equiv 0 \pmod 5$.
- 9: $x \equiv 0 \pmod 9$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod 9$.

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv 0 \pmod 2$ iff $x_0 \equiv 0 \pmod 2$.
- 3: $x \equiv 0 \pmod 3$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod 3$.
- 5: $x \equiv 0 \pmod 5$ iff $x_0 \equiv 0 \pmod 5$.
- 9: $x \equiv 0 \pmod 9$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod 9$.
- 10:

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.

Let $x = x_n \cdots x_0$.

- 2: $x \equiv 0 \pmod{2}$ iff $x_0 \equiv 0 \pmod{2}$.
- 3: $x \equiv 0 \pmod{3}$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod{3}$.
- 5: $x \equiv 0 \pmod{5}$ iff $x_0 \equiv 0 \pmod{5}$.
- 9: $x \equiv 0 \pmod{9}$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod{9}$.
- 10: $x \equiv 0 \pmod{10}$ iff $x_0 \equiv 0 \pmod{10}$.

What I Learned in Junior High School

Divisibility tricks for 2,3,5,9,10.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv 0 \pmod{2}$ iff $x_0 \equiv 0 \pmod{2}$.
- 3: $x \equiv 0 \pmod{3}$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod{3}$.
- 5: $x \equiv 0 \pmod{5}$ iff $x_0 \equiv 0 \pmod{5}$.
- 9: $x \equiv 0 \pmod{9}$ iff is $\sum_{i=0}^{n} x_i \equiv 0 \pmod{9}$.
- 10: $x \equiv 0 \pmod{10}$ iff $x_0 \equiv 0 \pmod{10}$.

What is a trick? We come back to that later.

What I Didn't Learned in Junior High School

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

- 2:

What I Didn't Learned in Junior High School

We don't just learn divisibility.

Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod 2$

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod 2$
- 3:

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod 2$
- 3: $x \equiv \sum_{i=0}^{n} x_i \pmod 3$

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod{2}$
- 3: $x \equiv \sum_{i=0}^{n} x_i \pmod{3}$
- 5:

What I Didn't Learned in Junior High School

We don't just learn divisibility.

Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod 2$
- 3: $x \equiv \sum_{i=0}^{n} x_i \pmod 3$
- 5: $x \equiv x_0 \pmod 5$

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod 2$
- 3: $x \equiv \sum_{i=0}^{n} x_i \pmod 3$
- 5: $x \equiv x_0 \pmod 5$
- 9:

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod 2$
- 3: $x \equiv \sum_{i=0}^{n} x_i \pmod 3$
- 5: $x \equiv x_0 \pmod 5$
- 9: $x \equiv \sum_{i=0}^{n} x_i \pmod 9$

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod 2$
- 3: $x \equiv \sum_{i=0}^{n} x_i \pmod 3$
- 5: $x \equiv x_0 \pmod 5$
- 9: $x \equiv \sum_{i=0}^{n} x_i \pmod 9$
- 10:

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod 2$
- 3: $x \equiv \sum_{i=0}^{n} x_i \pmod 3$
- 5: $x \equiv x_0 \pmod 5$
- 9: $x \equiv \sum_{i=0}^{n} x_i \pmod 9$
- 10: $x \equiv x_0 \pmod{10}$

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod{2}$
- 3: $x \equiv \sum_{i=0}^{n} x_i \pmod{3}$
- 5: $x \equiv x_0 \pmod{5}$
- 9: $x \equiv \sum_{i=0}^{n} x_i \pmod{9}$
- 10: $x \equiv x_0 \pmod{10}$

1) We don't just get divisibility, we get **mod**.

What I Didn't Learned in Junior High School

We don't just learn divisibility.
Let $x = x_n \cdots x_0$.

- 2: $x \equiv x_0 \pmod{2}$
- 3: $x \equiv \sum_{i=0}^{n} x_i \pmod{3}$
- 5: $x \equiv x_0 \pmod{5}$
- 9: $x \equiv \sum_{i=0}^{n} x_i \pmod{9}$
- 10: $x \equiv x_0 \pmod{10}$

1) We don't just get divisibility, we get **mod**.

2) Still have not defined **trick** carefully.

Notation For this Slide Packet

For this Slide Packet $\Sigma = \{0, \ldots, 9\}$.

Notation For this Slide Packet

For this Slide Packet $\Sigma = \{0, \ldots, 9\}$.

Strings are numbers in base 10.

Notation For this Slide Packet

For this Slide Packet $\Sigma = \{0, \ldots, 9\}$.

Strings are numbers in base 10.

The string

$$d_{n-1} \cdots d_0$$

Notation For this Slide Packet

For this Slide Packet $\Sigma = \{0, \ldots, 9\}$.

Strings are numbers in base 10.

The string

$$d_{n-1} \cdots d_0$$

is the number

$$d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10^1 + d_0 \times 10^0.$$

Notation For this Slide Packet

For this Slide Packet $\Sigma = \{0, \ldots, 9\}$.

Strings are numbers in base 10.

The string

$$d_{n-1} \cdots d_0$$

is the number

$$d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10^1 + d_0 \times 10^0.$$

We feed a number into a DFA right-to-left:

d_0, then d_1 then d_2 then ....

Proof of Trick for Mod. All $\equiv$ are mod 2.

Proof of Trick for Mod. All $\equiv$ are mod 2.

Thm $d_{n-1} \cdots d_0 \equiv d_0$.

Proof of Trick for Mod. All $\equiv$ are mod 2.

Thm $d_{n-1} \cdots d_0 \equiv d_0$.

Pf

$$d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0$$

Proof of Trick for Mod. All $\equiv$ are mod 2.

Thm $d_{n-1} \cdots d_0 \equiv d_0$.

Pf

$$\begin{aligned} & d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \\ = \ & 10(d_{n-1} \times 10^{n-2} + \cdots + d_1) + d_0 \end{aligned}$$

Proof of Trick for Mod. All $\equiv$ are mod 2.

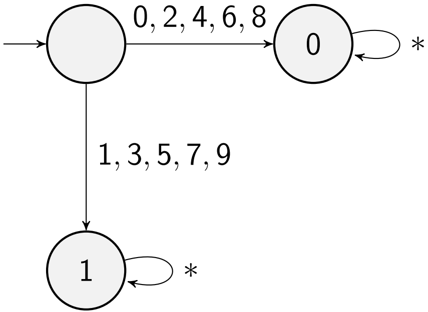
Thm $d_{n-1} \cdots d_0 \equiv d_0$.

Pf

$$\begin{aligned} & d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \\ = \;& 10(d_{n-1} \times 10^{n-2} + \cdots + d_1) + d_0 \\ \equiv \;& d_0 \end{aligned}$$

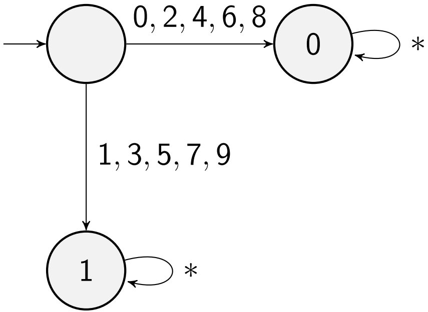
DFA Classifier for Mod 2

DFA Classifier for Mod 2



We don't regard the empty string as even or odd.

DFA Classifier for Mod 2



We don't regard the empty string as even or odd.
If pressed, I would say even.

Proof of Trick for Mod 3. All $\equiv$ are mod 3.

Thm $d_{n-1} \cdots d_0 \equiv d_{n-1} + \cdots + d_0$.

Proof of Trick for Mod 3. All $\equiv$ are mod 3.

Thm $d_{n-1} \cdots d_0 \equiv d_{n-1} + \cdots + d_0$.

Pf

$$d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \times 10^0$$

Proof of Trick for Mod 3. All $\equiv$ are mod 3.

Thm $d_{n-1} \cdots d_0 \equiv d_{n-1} + \cdots + d_0$.

Pf

$$\begin{aligned} & d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \times 10^0 \\ \equiv \quad & d_{n-1} \times 1 + \cdots + d_1 \times 1 + d_0 \times 1 \end{aligned}$$

Proof of Trick for Mod 3. All $\equiv$ are mod 3.

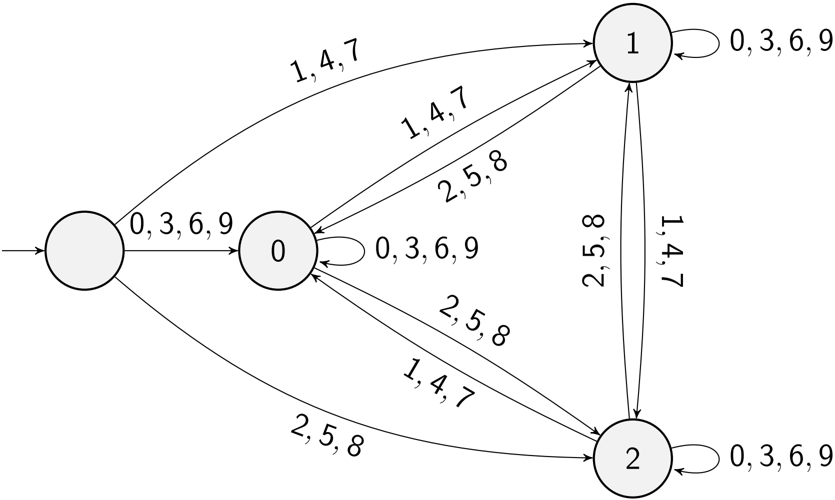
Thm $d_{n-1} \cdots d_0 \equiv d_{n-1} + \cdots + d_0$.

Pf

$$
\begin{aligned}
& d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \times 10^0 \\
\equiv \quad & d_{n-1} \times 1 + \cdots + d_1 \times 1 + d_0 \times 1 \\
\equiv \quad & d_{n-1} + \cdots + d_1 + d_0
\end{aligned}
$$

DFA Classifier for Mod 3

DFA Classifier for Mod 3



Trick for Mod 4. All $\equiv$ are Mod 4

Do you know the Mod 4 trick??

Trick for Mod 4. All $\equiv$ are Mod 4

Do you know the Mod 4 trick??

$n \equiv 0$ iff

Trick for Mod 4. All $\equiv$ are Mod 4

Do you know the Mod 4 trick??
$n \equiv 0$ iff last 2 digits are a number $\equiv 0$.

Trick for Mod 4. All $\equiv$ are Mod 4

Do you know the Mod 4 trick??
$n \equiv 0$ iff last 2 digits are a number $\equiv 0$.

Thm $d_{n-1} \cdots d_0 \equiv 2d_1 + d_0$.

Trick for Mod 4. All $\equiv$ are Mod 4

Do you know the Mod 4 trick??
$n \equiv 0$ iff last 2 digits are a number $\equiv 0$.

Thm $d_{n-1} \cdots d_0 \equiv 2d_1 + d_0$.

Pf

$$d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0$$

Trick for Mod 4. All $\equiv$ are Mod 4

Do you know the Mod 4 trick??

$n \equiv 0$ iff last 2 digits are a number $\equiv 0$.

Thm $d_{n-1} \cdots d_0 \equiv 2d_1 + d_0$.

Pf

$$\begin{aligned} & d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \\ \equiv \; & d_1 \times 10 + d_0 \end{aligned}$$

Trick for Mod 4. All $\equiv$ are Mod 4

Do you know the Mod 4 trick??

$n \equiv 0$ iff last 2 digits are a number $\equiv 0$.

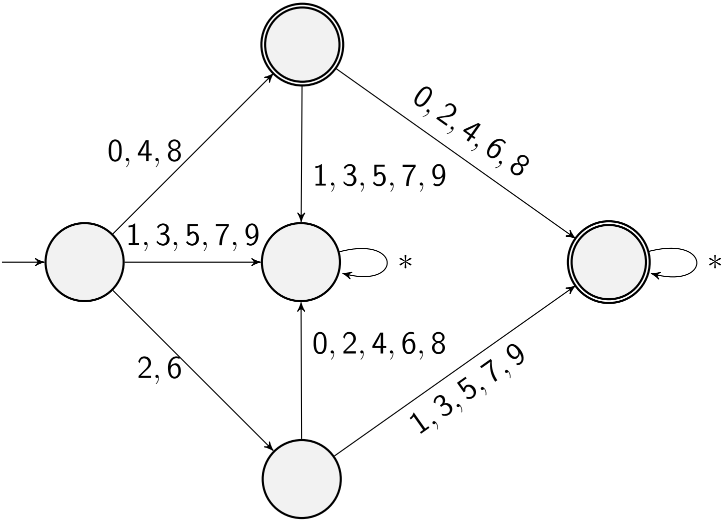
Thm $d_{n-1} \cdots d_0 \equiv 2d_1 + d_0$.

Pf

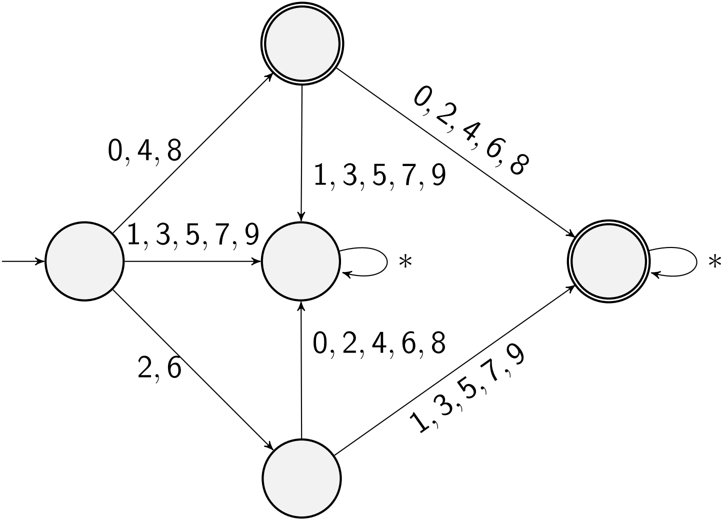
$$\begin{aligned} & d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \\ \equiv \;& d_1 \times 10 + d_0 \\ \equiv \;& 2d_1 + d_0. \end{aligned}$$

DFA for Mod 4 that just says $\equiv 0$ OR NOT

DFA for Mod 4 that just says $\equiv 0$ OR NOT

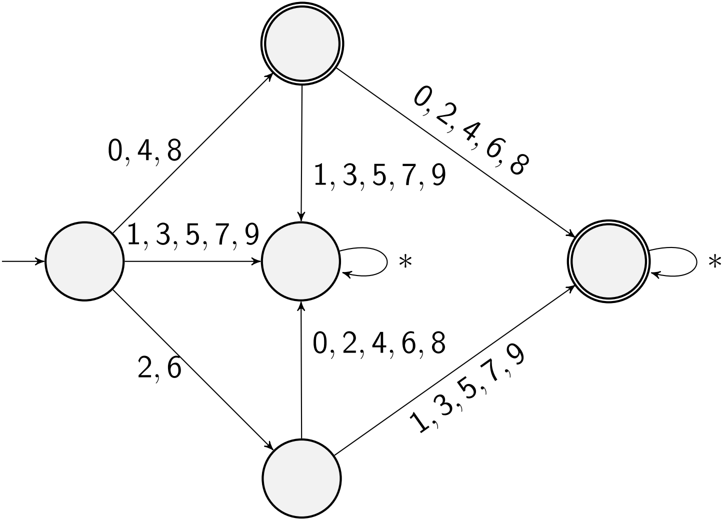


DFA for Mod 4 that just says $\equiv 0$ OR NOT



This DFA determines $\equiv 0 \pmod 4$ or NOT.

DFA for Mod 4 that just says $\equiv 0$ OR NOT



This DFA determines $\equiv 0 \pmod 4$ or NOT.
A DFA classifier for mod 4 may be on the HW.

Key to all of these Problems

For all of these problems we need to find a pattern of $10^n \pmod{a}$.

Key to all of these Problems

For all of these problems we need to find a pattern of $10^n \pmod{a}$.

Mod 2: Pattern is 1,0,0,0,..., DFA cared about first digit.

Key to all of these Problems

For all of these problems we need to find a pattern of $10^n \pmod{a}$.
Mod 2: Pattern is 1,0,0,0,..., DFA cared about first digit.
Mod 3: Pattern is 1,1,1,1,..., DFA tracked sum mod 3.

Key to all of these Problems

For all of these problems we need to find a pattern of $10^n \pmod{a}$.
Mod 2: Pattern is 1,0,0,0,. . ., DFA cared about first digit.
Mod 3: Pattern is 1,1,1,1,. . ., DFA tracked sum mod 3.
Mod 4: Pattern is 1,2,0,0,0,. . ., DFA cared about first 2 digits.

Proof of Tricks for Mod 5,9,10 and Trick for Mod 6

These may be on a HW.

Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?

Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?
We derive it together!

Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?
We derive it together!
$10^0 \equiv 1$

Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?
We derive it together!
$10^0 \equiv 1$
$10^1 \equiv 10 \equiv -1$

Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?
We derive it together!
$10^0 \equiv 1$
$10^1 \equiv 10 \equiv -1$
$10^2 \equiv 10 \times 10 \equiv -1 \times -1 \equiv 1.$

Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?
We derive it together!
$10^0 \equiv 1$
$10^1 \equiv 10 \equiv -1$
$10^2 \equiv 10 \times 10 \equiv -1 \times -1 \equiv 1.$
$10^3 \equiv 10^2 \times 10 \equiv 1 \times -1 \equiv -1.$
Pattern is $1, -1, 1, -1, \ldots.$

Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?
We derive it together!
$10^0 \equiv 1$
$10^1 \equiv 10 \equiv -1$
$10^2 \equiv 10 \times 10 \equiv -1 \times -1 \equiv 1.$
$10^3 \equiv 10^2 \times 10 \equiv 1 \times -1 \equiv -1.$
Pattern is $1, -1, 1, -1, \ldots$.
Thm $d_n \cdots d_0 \equiv d_0 - d_1 + d_2 - \cdots \pm d_n.$

Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?
We derive it together!
$10^0 \equiv 1$
$10^1 \equiv 10 \equiv -1$
$10^2 \equiv 10 \times 10 \equiv -1 \times -1 \equiv 1.$
$10^3 \equiv 10^2 \times 10 \equiv 1 \times -1 \equiv -1.$
Pattern is $1, -1, 1, -1, \ldots.$

Thm $d_n \cdots d_0 \equiv d_0 - d_1 + d_2 - \cdots \pm d_n.$

Proof may be on HW or Midterm or Final or some combination.

DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

$Q = \{0, \ldots, 10\} \times \{0, 1\}$

DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

$Q = \{0, \ldots, 10\} \times \{0, 1\}$

$s = (0, 0)$.

DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

$Q = \{0, \ldots, 10\} \times \{0, 1\}$

$s = (0, 0)$.

Final state: Not going to have these, this is DFA-classifier.

DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

$Q = \{0, \ldots, 10\} \times \{0, 1\}$

$s = (0, 0)$.

Final state: Not going to have these, this is DFA-classifier.

$$\delta((i,j),\sigma) \begin{cases} (i+\sigma \pmod{11}, j+1 \pmod{2}) \text{ if } j=0 \\ (i-\sigma \pmod{11}, j+1 \pmod{2}) \text{ if } j=1 \end{cases} \tag{1}$$

DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

$Q = \{0, \ldots, 10\} \times \{0, 1\}$

$s = (0, 0)$.

Final state: Not going to have these, this is DFA-classifier.

$$\delta((i,j),\sigma) \begin{cases} (i+\sigma \pmod{11}, j+1 \pmod{2}) \text{ if } j=0 \\ (i-\sigma \pmod{11}, j+1 \pmod{2}) \text{ if } j=1 \end{cases} \tag{1}$$

We keep track of a running weighted sum mod 11 and position of the symbol mod 2.

DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

$Q = \{0, \ldots, 10\} \times \{0, 1\}$

$s = (0, 0)$.

Final state: Not going to have these, this is DFA-classifier.

$$\delta((i,j),\sigma) \begin{cases} (i+\sigma \pmod{11}, j+1 \pmod{2}) \text{ if } j=0 \\ (i-\sigma \pmod{11}, j+1 \pmod{2}) \text{ if } j=1 \end{cases} \tag{1}$$

We keep track of a running weighted sum mod 11 and position of the symbol mod 2.

22 states.

DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

$Q = \{0, \ldots, 10\} \times \{0, 1\}$

$s = (0, 0)$.

Final state: Not going to have these, this is DFA-classifier.

$$\delta((i,j),\sigma) \begin{cases} (i+\sigma \pmod{11}, j+1 \pmod{2}) \text{ if } j=0 \\ (i-\sigma \pmod{11}, j+1 \pmod{2}) \text{ if } j=1 \end{cases} \tag{1}$$

We keep track of a running weighted sum mod 11 and position of the symbol mod 2.

22 states.

Classifier If end in $(i, 0)$ or $(i, 1)$ then number is $\equiv i$.

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?
Answer Depends what you call a trick.

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?
Answer Depends what you call a trick.
We need to spot a pattern.

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?
Answer Depends what you call a trick.
We need to spot a pattern.

$10^0 \equiv 1$

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?

Answer Depends what you call a trick.

We need to spot a pattern.

$10^0 \equiv 1$

$10^1 \equiv 3$

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?
Answer Depends what you call a trick.
We need to spot a pattern.

$10^0 \equiv 1$

$10^1 \equiv 3$

$10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2$

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?
Answer Depends what you call a trick.
We need to spot a pattern.

$10^0 \equiv 1$

$10^1 \equiv 3$

$10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2$

$10^3 \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6$

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?
Answer Depends what you call a trick.
We need to spot a pattern.

$10^0 \equiv 1$

$10^1 \equiv 3$

$10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2$

$10^3 \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6$

$10^4 \equiv 10^3 \times 10 \equiv 6 \times 3 \equiv 18 \equiv 4$

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?

Answer Depends what you call a trick.

We need to spot a pattern.

$10^0 \equiv 1$

$10^1 \equiv 3$

$10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2$

$10^3 \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6$

$10^4 \equiv 10^3 \times 10 \equiv 6 \times 3 \equiv 18 \equiv 4$

$10^5 \equiv 10^4 \times 10 \equiv 4 \times 3 \equiv 12 \equiv 5$

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?
Answer Depends what you call a trick.
We need to spot a pattern.

$10^0 \equiv 1$

$10^1 \equiv 3$

$10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2$

$10^3 \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6$

$10^4 \equiv 10^3 \times 10 \equiv 6 \times 3 \equiv 18 \equiv 4$

$10^5 \equiv 10^4 \times 10 \equiv 4 \times 3 \equiv 12 \equiv 5$

$10^6 \equiv 10^5 \times 10 \equiv 5 \times 3 \equiv 15 \equiv 1$

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?
Answer Depends what you call a trick.
We need to spot a pattern.

$10^0 \equiv 1$

$10^1 \equiv 3$

$10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2$

$10^3 \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6$

$10^4 \equiv 10^3 \times 10 \equiv 6 \times 3 \equiv 18 \equiv 4$

$10^5 \equiv 10^4 \times 10 \equiv 4 \times 3 \equiv 12 \equiv 5$

$10^6 \equiv 10^5 \times 10 \equiv 5 \times 3 \equiv 15 \equiv 1$

Pattern is $1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, \ldots$.

Is There a Trick for Mod 7? All $\equiv$ are Mod 7

Is there a trick for mod 7?
Answer Depends what you call a trick.
We need to spot a pattern.

$10^0 \equiv 1$

$10^1 \equiv 3$

$10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2$

$10^3 \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6$

$10^4 \equiv 10^3 \times 10 \equiv 6 \times 3 \equiv 18 \equiv 4$

$10^5 \equiv 10^4 \times 10 \equiv 4 \times 3 \equiv 12 \equiv 5$

$10^6 \equiv 10^5 \times 10 \equiv 5 \times 3 \equiv 15 \equiv 1$

Pattern is $1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, \ldots$.
Can we use this?

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

$$\begin{aligned} & 3876554 \\ = \; & 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \end{aligned}$$

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

$$
\begin{aligned}
& 3876554 \\
= \;& 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \\
\equiv \;& 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod 7
\end{aligned}
$$

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

$$\begin{aligned}
& 3876554 \\
= \; & 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \\
\equiv \; & 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod 7 \\
\equiv \; & 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + -1 \cdot 6 + -2 \cdot 2 + -2 \cdot 3 + 4 \pmod 7
\end{aligned}$$

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

$$
\begin{aligned}
& 3876554 \\
= \;& 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \\
\equiv \;& 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod 7 \\
\equiv \;& 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + -1 \cdot 6 + -2 \cdot 2 + -2 \cdot 3 + 4 \pmod 7 \\
\equiv \;& 3 + 5 + 0 - 6 - 4 - 6 + 4 \pmod 7
\end{aligned}
$$

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

$$
\begin{aligned}
& 3876554 \\
= \;& 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \\
\equiv \;& 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod 7 \\
\equiv \;& 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + -1 \cdot 6 + -2 \cdot 2 + -2 \cdot 3 + 4 \pmod 7 \\
\equiv \;& 3 + 5 + 0 - 6 - 4 - 6 + 4 \pmod 7 \\
\equiv \;& 3 \pmod 7
\end{aligned}
$$

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

$$
\begin{aligned}
& 3876554 \\
= \;& 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \\
\equiv \;& 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod 7 \\
\equiv \;& 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + -1 \cdot 6 + -2 \cdot 2 + -2 \cdot 3 + 4 \pmod 7 \\
\equiv \;& 3 + 5 + 0 - 6 - 4 - 6 + 4 \pmod 7 \\
\equiv \;& 3 \pmod 7
\end{aligned}
$$

DFA States will keep track of

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

$$
\begin{aligned}
& 3876554 \\
= \ & 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \\
\equiv \ & 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod 7 \\
\equiv \ & 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + -1 \cdot 6 + -2 \cdot 2 + -2 \cdot 3 + 4 \pmod 7 \\
\equiv \ & 3 + 5 + 0 - 6 - 4 - 6 + 4 \pmod 7 \\
\equiv \ & 3 \pmod 7
\end{aligned}
$$

DFA States will keep track of

Running weighted sum mod 7

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

$$
\begin{aligned}
& 3876554 \\
= \;& 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \\
\equiv \;& 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod 7 \\
\equiv \;& 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + -1 \cdot 6 + -2 \cdot 2 + -2 \cdot 3 + 4 \pmod 7 \\
\equiv \;& 3 + 5 + 0 - 6 - 4 - 6 + 4 \pmod 7 \\
\equiv \;& 3 \pmod 7
\end{aligned}
$$

DFA States will keep track of

Running weighted sum mod 7

Position of digit mod 6 so know which weights to use.

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

$$
\begin{aligned}
& 3876554 \\
= \;& 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4 \\
\equiv \;& 3 \cdot 1 + 8 \cdot 5 + 7 \cdot 4 + 6 \cdot 6 + 5 \cdot 2 + 5 \cdot 3 + 4 \pmod 7 \\
\equiv \;& 3 \cdot 1 + 1 \cdot 5 + 0 \cdot 4 + -1 \cdot 6 + -2 \cdot 2 + -2 \cdot 3 + 4 \pmod 7 \\
\equiv \;& 3 + 5 + 0 - 6 - 4 - 6 + 4 \pmod 7 \\
\equiv \;& 3 \pmod 7
\end{aligned}
$$

DFA States will keep track of

Running weighted sum mod 7

Position of digit mod 6 so know which weights to use.

So there are $7 \times 6 = 42$ states.

Is the Method a Trick?

Is the Method a Trick?

YES A DFA can do it.

Is the Method a Trick?

YES A DFA can do it.

NO A human cannot do it easily. (The pattern is not like 1,1,1,... or mostly 0's.)

The DFA for $\{n : n \equiv 0 \pmod{7}\}$

The DFA for $\{n : n \equiv 0 \pmod 7\}$

Too hard for me ...

The DFA for $\{n : n \equiv 0 \pmod 7\}$

Too hard for me ...

... but not for you.

The DFA for $\{n : n \equiv 0 \pmod 7\}$

Too hard for me ...

... but not for you.

Might make it a HW to do as a table.

Possible Research Question

What is the fastest way to determine $n \pmod 7$?

Possible Research Question

What is the fastest way to determine $n \pmod 7$?

Method One Divide and take remainder.

Possible Research Question

What is the fastest way to determine $n \pmod 7$?

Method One Divide and take remainder.

Method Two Use the DFA.

Possible Research Question

What is the fastest way to determine $n \pmod 7$?

Method One Divide and take remainder.

Method Two Use the DFA.

Question Which is faster?

Possible Research Question

What is the fastest way to determine $n \pmod 7$?

Method One Divide and take remainder.

Method Two Use the DFA.

Question Which is faster?

Might be hard to tell because today's computers are so fast!