Tricks for Divisibility and DFA's

What I Learned in Junior High School

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What is a trick? We come back to that later.

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1) We don't just get divisibility, we get **mod**.

2) Still have not defined **trick** carefully.

Notation For this Slide Packet

For this Slide Packet $\Sigma = \{0, \ldots, 9\}$.

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We feed a number into a DFA right-to-left:

d_0, then d_1 then d_2 then ....

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$$\begin{aligned} & d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \\ = \; & 10(d_{n-1} \times 10^{n-2} + \cdots + d_1) + d_0 \end{aligned}$$

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DFA for Mod 2

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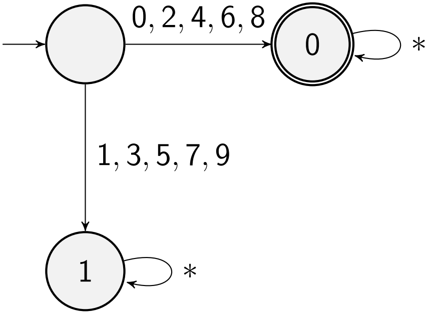

Proof of Trick for Mod 3. All $\equiv$ are mod 3.

Thm $d_{n-1}\cdots d_0 \equiv d_{n-1}+\cdots+d_0$.

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Thm $d_{n-1} \cdots d_0 \equiv d_{n-1} + \cdots + d_0$.

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Thm $d_{n-1} \cdots d_0 \equiv d_{n-1} + \cdots + d_0$.

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$$d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \times 10^0$$

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Thm $d_{n-1} \cdots d_0 \equiv d_{n-1} + \cdots + d_0$.

Pf

$$
\begin{aligned}
& d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10 + d_0 \times 10^0 \\
\equiv \; & d_{n-1} \times 1 + \cdots + d_1 \times 1 + d_0 \times 1
\end{aligned}
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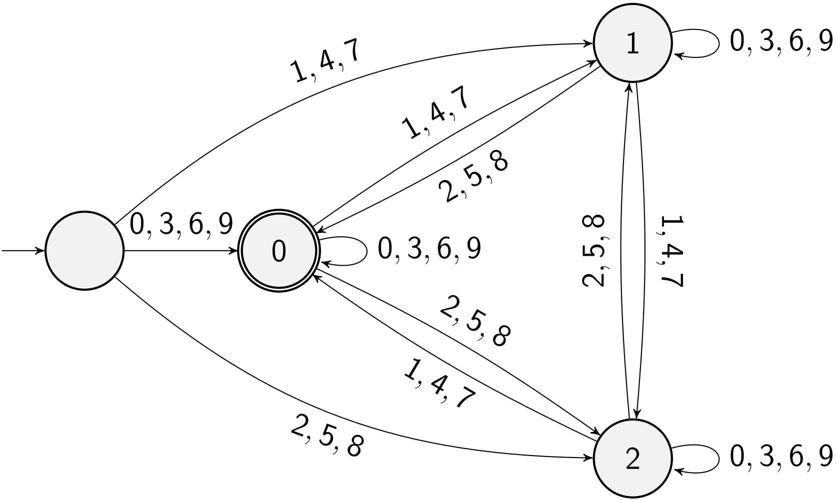

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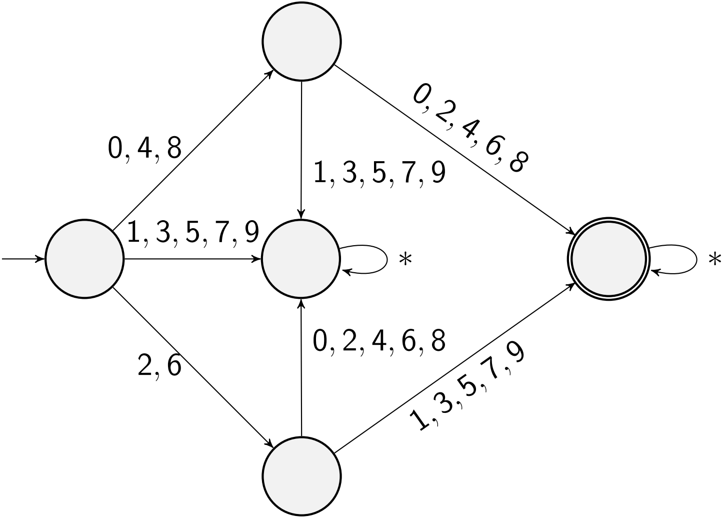

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For all of these problems we need to find a pattern of $10^n \pmod{a}$.

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Mod 4: Pattern is 1,2,0,0,0,..., DFA cared about first 2 digits.

Proof of Tricks for Mod 5,9,10 and Trick for Mod 6

These may be on a HW.

Trick for Mod 11. All $\equiv$ are Mod 11

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Pattern is $1, -1, 1, -1, \ldots$.

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Thm $d_n \cdots d_0 \equiv d_0 - d_1 + d_2 - \cdots \pm d_n.$

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Proof may be on HW or Midterm or Final or some combination.

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$$\delta((i,j),\sigma) \begin{cases} (i+\sigma \pmod{11}, j+1 \pmod{2}) \text{ if } j=0 \\ (i-\sigma \pmod{11}, j+1 \pmod{2}) \text{ if } j=1 \end{cases} \tag{1}$$

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We keep track of a running weighted sum mod 11 and position of the symbol mod 2.

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Classifier If end in $(i, 0)$ or $(i, 1)$ then number is $\equiv i$.

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$10^6 \equiv 10^5 \times 10 \equiv 5 \times 3 \equiv 15 \equiv 1$

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Pattern is $1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, \ldots$.

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$10^3 \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6$

$10^4 \equiv 10^3 \times 10 \equiv 6 \times 3 \equiv 18 \equiv 4$

$10^5 \equiv 10^4 \times 10 \equiv 4 \times 3 \equiv 12 \equiv 5$

$10^6 \equiv 10^5 \times 10 \equiv 5 \times 3 \equiv 15 \equiv 1$

Pattern is $1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, \ldots$.
Can we use this?

Using the Divide by 7 Trick

Want to know what 3876554 is mod 7.

Using the Divide by 7 Trick

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$$
\begin{aligned}
& 3876554 \\
= \quad & 3 \cdot 10^6 + 8 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 5 \cdot 10 + 4
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DFA States will keep track of

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DFA States will keep track of

Running weighted sum mod 7

Using the Divide by 7 Trick

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DFA States will keep track of

Running weighted sum mod 7

Position of digit mod 6 so know which weights to use.

Using the Divide by 7 Trick

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DFA States will keep track of

Running weighted sum mod 7

Position of digit mod 6 so know which weights to use.

So there are $7 \times 6 = 42$ states.

Is the Method a Trick?

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YES A DFA can do it.

Is the Method a Trick?

YES A DFA can do it.

NO A human cannot do it easily. (The pattern is not like 1,1,1,. . . or mostly 0's.)

The DFA for $\{n : n \equiv 0 \pmod 7\}$

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Too hard for me ...

The DFA for $\{n : n \equiv 0 \pmod 7\}$

Too hard for me ...

... but not for you.

The DFA for $\{n : n \equiv 0 \pmod{7}\}$

Too hard for me ...

... but not for you.

Might make it a HW to do as a table.

Possible Research Question

What is the fastest way to determine $n \pmod{7}$?

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What is the fastest way to determine $n \pmod 7$?

Method One Divide and take remainder.

Possible Research Question

What is the fastest way to determine $n \pmod 7$?

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Method Two Use the DFA.

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Question Which is faster?

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Might be hard to tell because today's computers are so fast!