Deterministic Finite Automata (DFA)

DFAs

DFAs

Three Examples

Standard Conventions

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2. The states that are circled are **final states**. If the machine ends up there, then the string is accepted.

DFA Diagram: A First Example

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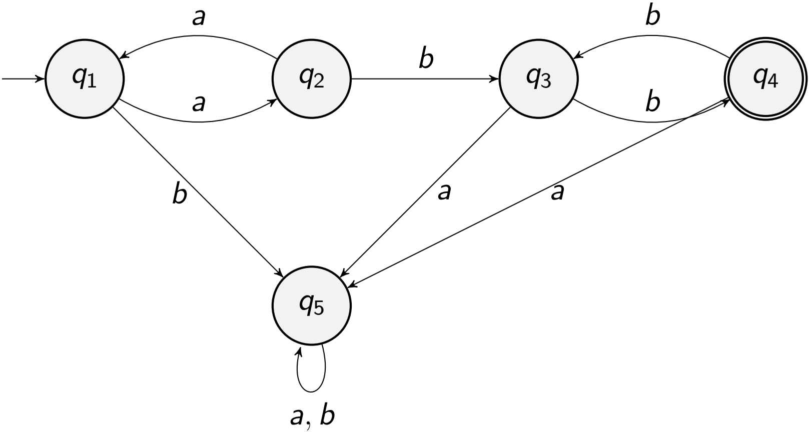

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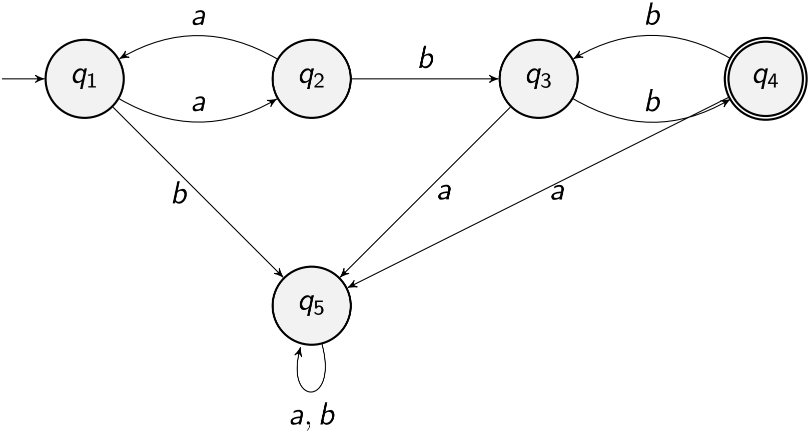

What is the language?

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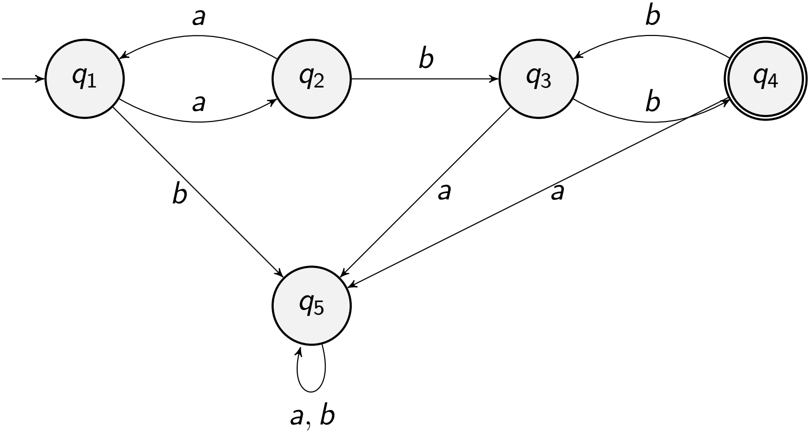

What is the language?

Odd number of a's followed by an even number of b's, but at least two.

DFA Diagram: A Second Example

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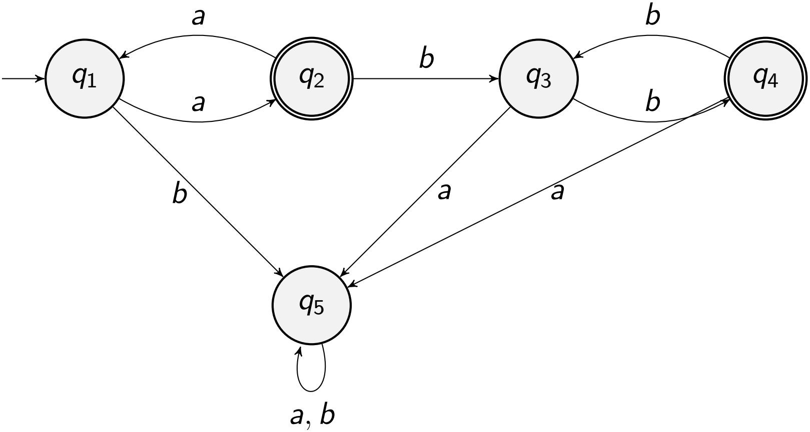

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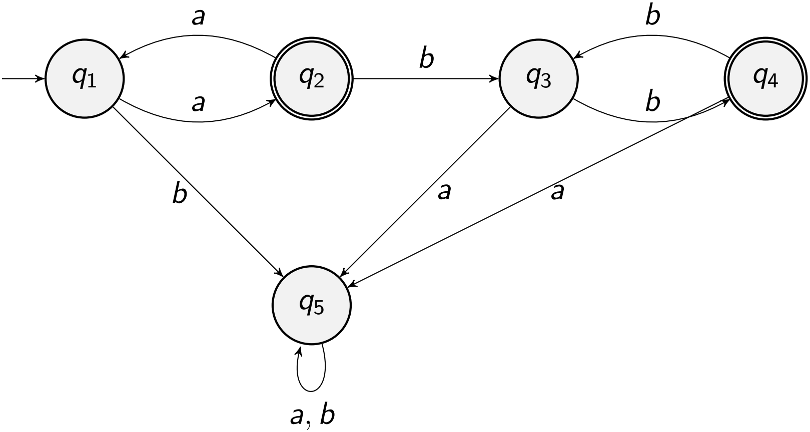

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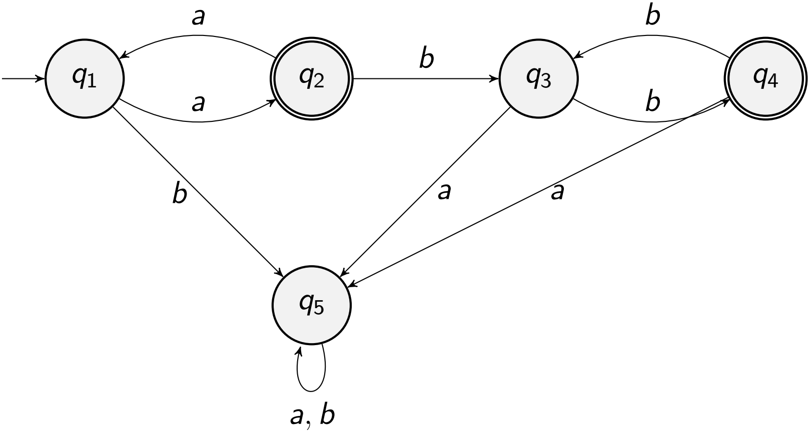

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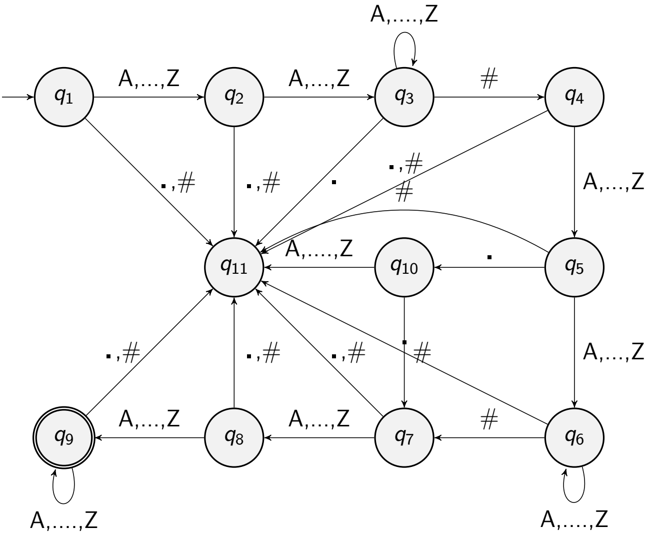

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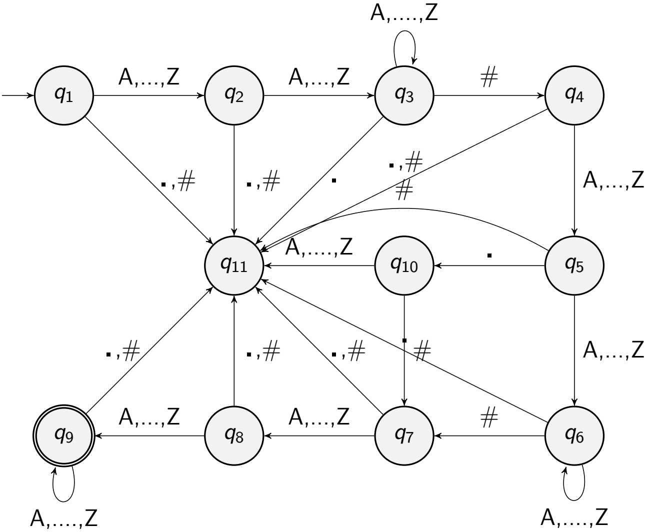

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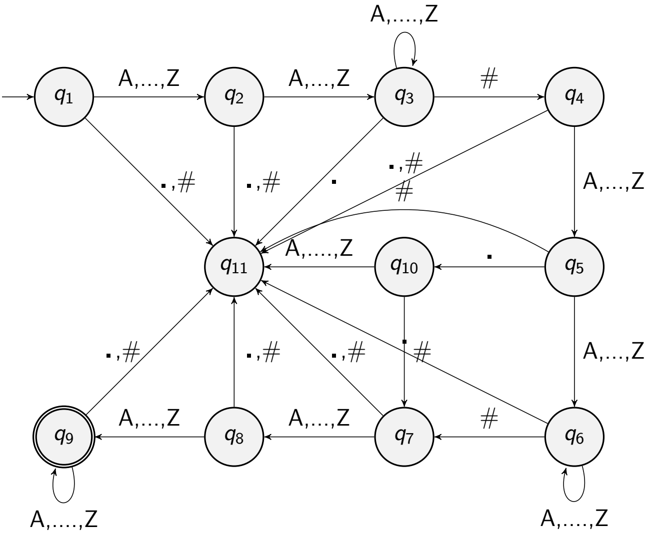

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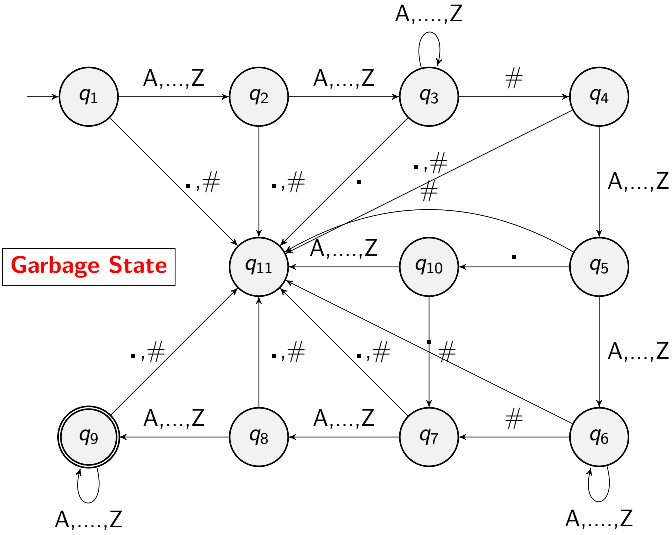

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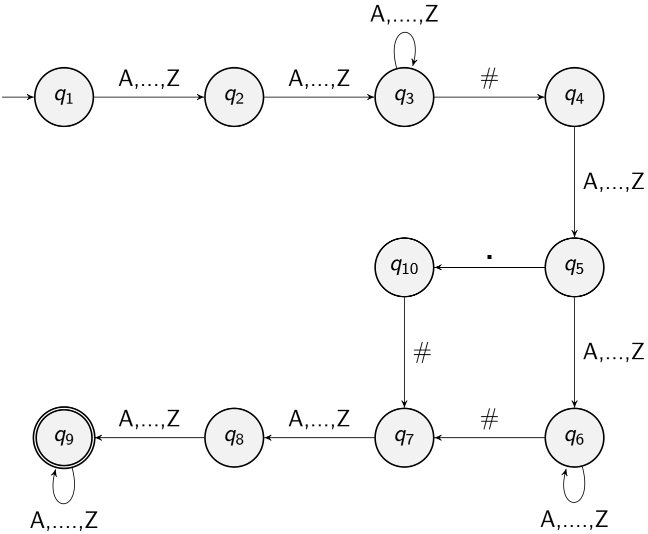

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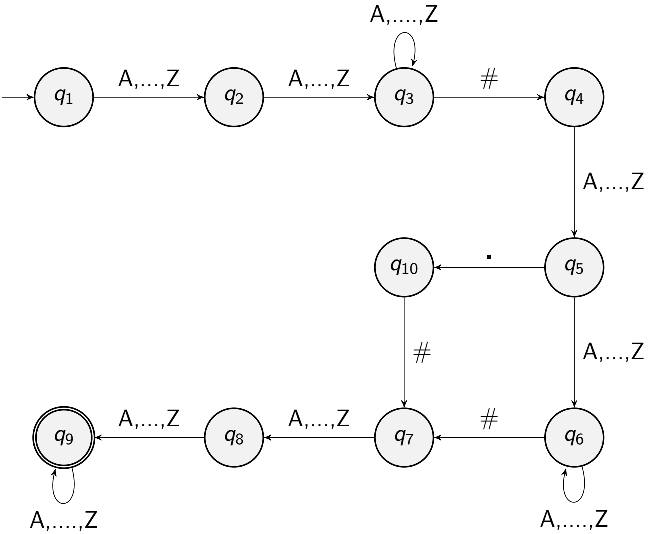

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Short Detour

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Modular Arithmetic

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- $100 \equiv 2 \pmod{7}$ since $100 = 7 \times 14 + 2$.

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When dealing with mod n we assume the entire universe is $\{0, 1, \ldots, n-1\}$.

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$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

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4. Division: Next Slide.

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Fact: A number y has an inverse mod 26 if y and 26 have no common factors. Numbers that have an inverse mod 26:

$$\{1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25\}$$

End of Detour

End of Detour

Another Example

$$\{w : \#_a(w) \equiv 1 \pmod 2 \wedge \#_b(w) \equiv 2 \pmod 3\}$$

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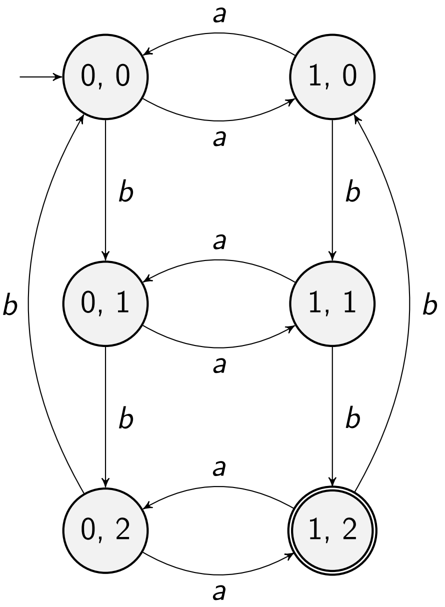

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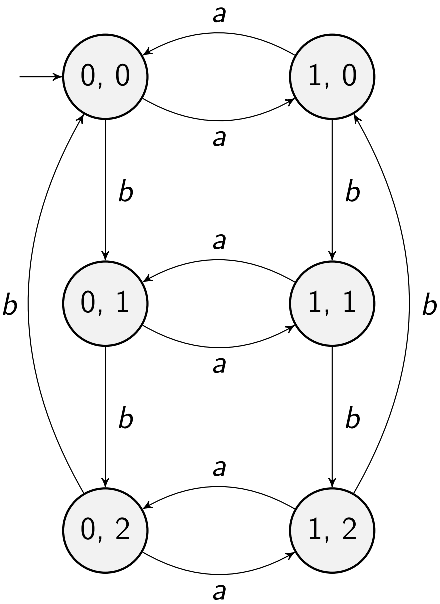

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The second DFA **classifies** strings without judgment.

Short Detour

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Alphabets, Strings, and Languages

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- $i = 1$ case is just $\Sigma^1 = \Sigma$.
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- **Notation Kleene star:** $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \cdots$ is the set of all strings over the alphabet Σ (including e).

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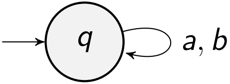

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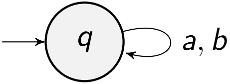

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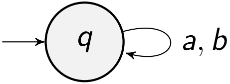

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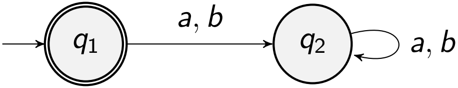

End of Detour

End of Detour

Start of Transition Tables

Recall Second Example

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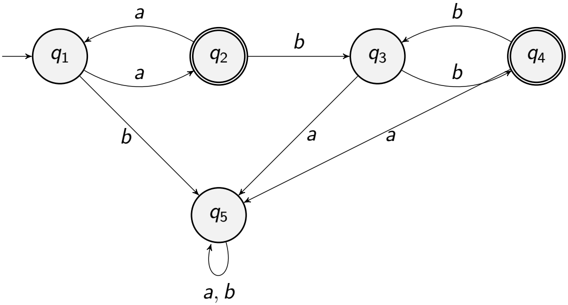

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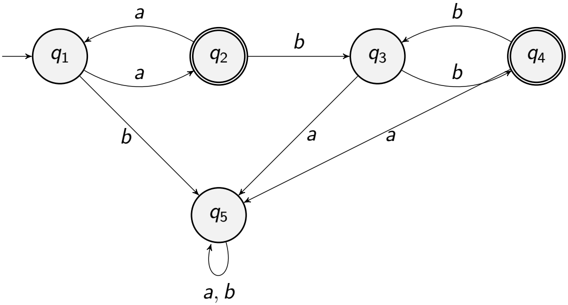

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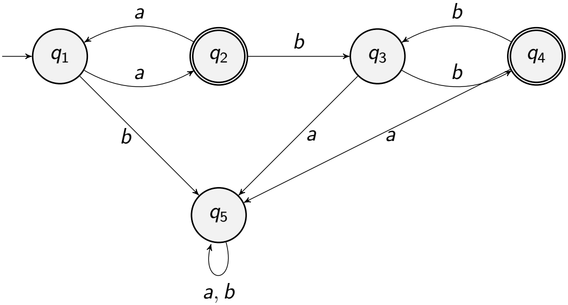

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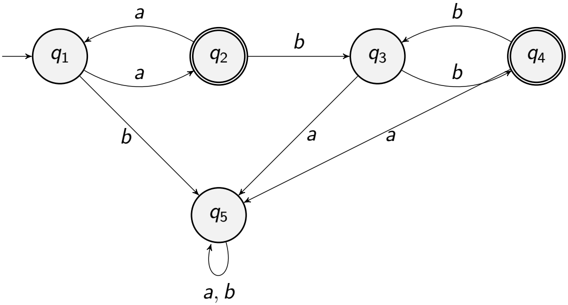

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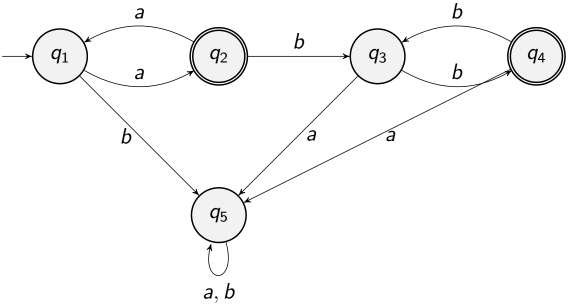

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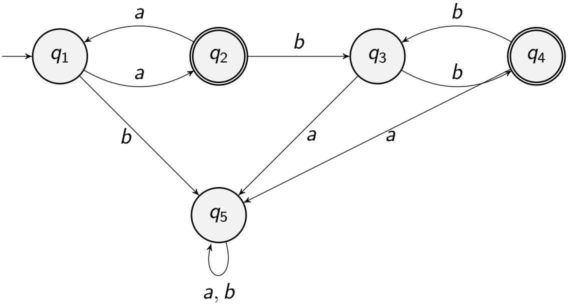

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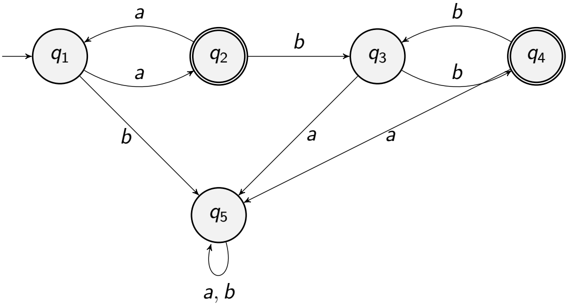

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Formal definition of DFAs

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Def A **DFA** M is a 5-tuple $(Q, \Sigma, \delta, s, F)$ where:

1. Q is a finite set of **states**.
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Def If M is a DFA and $x \in \Sigma^*$ is a word of length n, so $x = x_1 \cdots x_n$ where $x_i \in \Sigma$.
M* accepts *x if there is a sequence of states $q_0, q_1, \ldots, q_n$ such that $q_0 = s$, $q_i = \delta(q_{i-1}, x_i)$ for $1 \le i \le n$, and $q_n \in F$.

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Def Language $L \subseteq \Sigma^*$ is **regular** if there exists a DFA M such that $L(M) = L$.

Computer Implementation of DFAs

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Implementation of Transition Table:

- States: $\{1, 2, 3, 4, 5\}$
- Alphabet: $\{1, 2\}$
- Start state: 1
- Final states: $\{2, 4\}$

- Transition function

	1	2
1	2	5
2	1	3
3	5	4
4	5	3
5	5	5

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Linear time!

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- Transition tables are good for algorithms and formal proofs.