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Cocke-Younger-Kasimi Algorithm

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Sakai (1962), Kasimi (1965), Younger (1967), Cocke/Schwartz (1970)

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We want the algorithm to halt in $p(n)$ time for some polynomial.
We will obtain $p(n) = O(n^3)$.

Omitting a Case

We will assume that $e \notin L$ and hence we do not have the rule

$$S \rightarrow e.$$

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Our proof can be modified to accommodate this case.

Plan for CFL $\subset$ P

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$$\mathrm{GEN}[1, n] = \{A : A \Rightarrow \sigma_1 \cdots \sigma_n\}$$

We are really asking: Is $S \in \mathrm{GEN}[1, n]$?

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All we want to know is:

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All we want to know is:

$$\text{is } S \text{ in } \mathrm{GEN}[1, n]?$$

We will solve a harder problem:

$$\text{Find } \mathrm{GEN}[1, n]$$

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For $i \leq j$ let

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Hence we will find $\mathrm{GEN}[1,n]$.

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Why solve this harder problem?

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Hence we will find if $S \in \mathrm{GEN}[1,n]$.

Why solve this harder problem?

We will use Dynamic programming so having some $\mathrm{GEN}[i,j]$ solved will help us solve later ones.

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$$\sigma_1 \cdots \sigma_{i-1} \overbrace{\sigma_i}^{A} \sigma_{i+1} \cdots \sigma_n$$

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$$\mathrm{GEN}[i, i] \;=\; \{A : A \rightarrow \sigma_i\}$$

Bottom Up View

$$\sigma_1 \cdots \sigma_{i-1} \overbrace{\sigma_i}^{A} \sigma_{i+1} \cdots \sigma_n$$

$$\mathrm{GEN}[i, i] \;=\; \{A : A \to \sigma_i\}$$

$$\sigma_1 \cdots \sigma_{i-1} \overbrace{\sigma_i}^{B} \overbrace{\sigma_{i+1}}^{C} \sigma_{i+2} \cdots \sigma_n$$

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$$\mathrm{GEN}[i,i+1] = \{A : A \rightarrow BC \ \wedge \ B \rightarrow \sigma_i \ \wedge \ C \rightarrow \sigma_{i+1}\}$$

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$$\begin{aligned} \mathrm{GEN}[i,i+1] &= \{A : A \rightarrow BC \;\wedge\; B \rightarrow \sigma_i \;\wedge\; C \rightarrow \sigma_{i+1}\} \\ &= \{A : A \rightarrow BC \\ &\qquad\quad \wedge\; B \in GEN[i,i] \;\wedge\; C \in GEN[i+1,i+1]\} \end{aligned}$$

Bottom Up View (Continued)

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$$\sigma_1 \cdots \sigma_{i-1} \overbrace{\sigma_i}^{B} \overbrace{\sigma_{i+1}\sigma_{i+2}}^{C} \sigma_{i+3} \cdots \sigma_n$$

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$$\begin{aligned} \mathrm{GEN}[i, i+2] = \{A : A \to BC \\ \wedge \ (\quad & (B \to \sigma_i \ \wedge \ C \Rightarrow \sigma_{i+1}\sigma_{i+2}) \\ \vee \ & (B \Rightarrow \sigma_i\sigma_{i+1} \ \wedge \ C \to \sigma_{i+2}))\} \end{aligned}$$

Bottom Up View (Continued)

$$\sigma_1 \cdots \sigma_{i-1} \overbrace{\sigma_i}^{B} \overbrace{\sigma_{i+1}\sigma_{i+2}}^{C} \sigma_{i+3} \cdots \sigma_n$$

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$$\begin{aligned}
\mathrm{GEN}[i, i+2] &= \{A : A \rightarrow BC \\
&\quad \wedge \ (\quad (B \rightarrow \sigma_i \ \wedge \ C \Rightarrow \sigma_{i+1}\sigma_{i+2}) \\
&\qquad \vee \ (B \Rightarrow \sigma_i\sigma_{i+1} \ \wedge \ C \rightarrow \sigma_{i+2}))\} \\
&= \{A : A \rightarrow BC \\
&\quad \wedge \ (\quad (B \in GEN[i, i] \ \wedge \ C \in GEN[i+1, i+2]) \\
&\qquad \vee \ (B \in GEN[i, i+1] \ \wedge \ C \in GEN[i+2, i+2]))\}
\end{aligned}$$

Bottom Up View (Continued)

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$$\sigma_1 \cdots \sigma_{i-1} \overbrace{\sigma_i}^{B} \overbrace{\sigma_{i+1}\sigma_{i+3}}^{C} \sigma_{i+4} \cdots \sigma_n$$

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GEN[i,i+3]

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$$\text{GEN}[i,j] \quad = \quad \bigcup_{i \le k < j} \{A : A \rightarrow BC \ \wedge \ B \Rightarrow \sigma_i \cdots \sigma_k \ \wedge \ C \Rightarrow \sigma_{k+1} \cdots \sigma_j\}$$

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The Algorithm

```
for i = 1 to n do
    for j = i to n do
        GEN[i,j] ← ∅

for i = 1 to n do
    for all rules A → σ_i do
        GEN[i,i] ← GEN[i,i] with A

for s = 2 to n do
    for i = 1 to n-s+1 do
        j ← i+s-1 do
        for k = i to j-1 do
            for all rules A → BC
                where B ∈ GEN[i,k] and C ∈ GEN[k+1,j]
                    GEN[i,j] ← GEN[i,j] with A
```