Converting a DFA to a REG EXP: An Example

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$M = (Q, \Sigma, \delta, s, F)$ is a DFA. $R(i, j, k)$ is a reg exp for $\{x \mid \overline{\delta}(i, x) = j\}$.

Recall:

$$R(i, j, 0) = \{\sigma \in \Sigma \mid \delta(i, \sigma) = j\}.$$

$$R(i, i, 0) = \{e\} \cup \{\sigma \in \Sigma \mid \delta(i, \sigma) = j\}.$$

$$R(i, j, k) = R(i, j, k-1) \cup R(i, k, k-1)R(k, k, k-1)^* R(k, j, k-1)$$

The regular expression for the language accepted by M is $\bigcup_{f \in F} R(1, f, n)$

We work this out on the following small example. We will start with an NFA rather than a DFA– The NFA will leave out some transitions and hence be smaller in this case, better to make the example shorter.

$Q = \{1, 2, 3\}$

$s = 1$

$F = \{3\}$

$\delta(1, a) = 2$

$\delta(1, b) = 3$

$\delta(2, a) = 3$

$\delta(2, b)$ does not exist.

$\delta(3, a) = 3$

$\delta(3, b) = 3$

We want to know $R(1, 3, 3)$. Rather than compute all $3 \times 3 \times 4 = 36$ $R(i, j, k)$'s, we see which ones we need.

ALL OF THE $R(-, -, 3)$ THAT WE NEED: $R(1, 3, 3)$. (only 1)

Since $R(1, 3, 3) = R(1, 3, 2) \cup R(1, 3, 2)R(3, 3, 2)^* R(3, 3, 2)$

ALL OF THE $R(-, -, 2)$ THAT WE NEED: $R(1, 3, 2), R(3, 3, 2)$. (only 2)

We need $R(1, 3, 2)$. We use

$$R(1, 3, 2) = R(1, 3, 1) \cup R(1, 2, 1)R(2, 2, 1)^* R(2, 3, 1)$$

Hence we need $R(1, 3, 1), R(1, 2, 1), R(2, 2, 1), R(2, 3, 1)$.

We need $R(3, 3, 2)$. ANOTHER SHORTCUT: Since state 3 is a self-loop it cannot ever use any other state, so $R(3, 3, 2) = R(3, 3, 0)$. We keep this in mind for later.

ALL OF THE $R(-, -, 1)$ THAT WE NEED:

$R(1, 2, 1), R(1, 3, 1), R(2, 2, 1), R(2, 3, 1)$ (only 4).

We are not going to bother to figure out which $R(-, -, 0)$ we need since its easier to just computer all nine of them. Note that we will get $R(3, 3, 0)$ which we need.

We first look at ALL of the $R(i, j, 0)$.

$R(1, 1, 0) = e$

$R(1,2,0) = a$
$R(1,3,0) = b$
$R(2,1,0) = \emptyset.$
$R(2,2,0) = e$
$R(2,3,0) = a$
$R(3,1,0) = \emptyset$
$R(3,2,0) = \emptyset$
$R(3,3,0) = e \cup a \cup b$

We now look at all of the $R(i,j,1)$ that we need.

$R(1,2,1) = R(1,2,0) \cup R(1,1,0)R(1,1,0)^* R(1,2,0) = a \cup ee^* a = a$
$R(1,3,1) = R(1,3,0) \cup R(1,1,0)R(1,1,0)^* R(1,3,0) = b \cup ee^* b = b$
$R(2,2,1) = R(2,2,0) \cup R(2,1,0)R(1,1,0)^* R(1,2,0) = e \cup \emptyset e^* a = e \cup \emptyset = e$
$R(2,3,1) = R(2,3,0) \cup R(2,1,0)R(1,1,0)^* R(1,3,0) = a \cup \emptyset e^* b = e \cup \emptyset = a$
We now look at all of the $R(i,j,2)$ that we need.

$R(1,3,2) = R(1,3,1) \cup R(1,2,1)R(2,2,1)^* R(2,3,1) = b \cup ae^* a = b \cup aa$
$R(3,3,2) = R(3,3,0) = e \cup a \cup b$

We now look at all of the $R(i,j,3)$, just $R(1,3,3)$.

$R(1,3,3) = R(1,3,2) \cup R(1,3,2)R(3,3,2)^* R(3,3,2) = (b \cup aa) \cup (b \cup aa)(e \cup a \cup b)^* (a \cup b).$

Reg Exp for the language is $R(1,3,3)$ above.