A Linear Time and Space Local Geometry Encoder via Vectorized Kernel Mixture (VecKM)

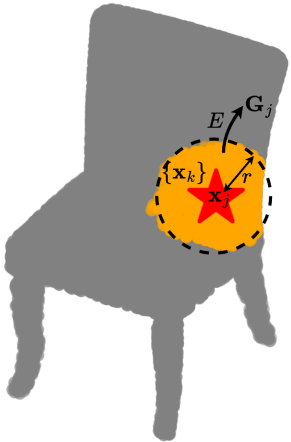
Dehao Yuan, Cornelia Fermüller, Yiannis Aloimonos

Department of Computer Science
University of Maryland, College Park

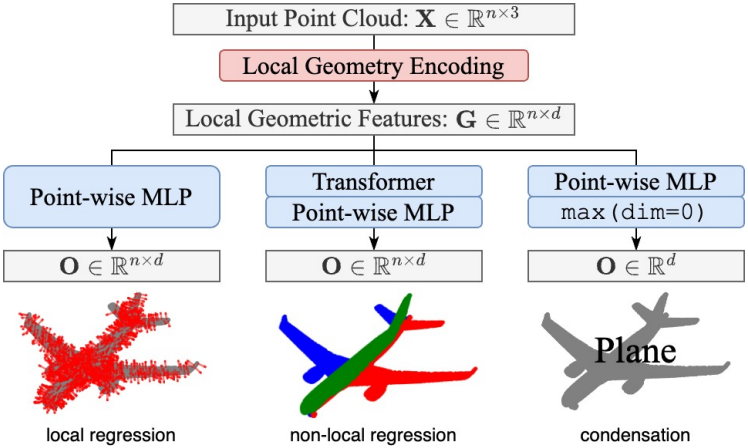
https://arxiv.org/abs/2404.01568
https://github.com/dhyuan99/VecKM
In 2024 International Conference of Machine Learning (ICML).

Local Geometry Encoding

- Input: point cloud $\mathbf{X}_{n\times 3}$.
- For each point $\mathbf{x}_j$, get the centerized neighborhood $\mathfrak{N}(\mathbf{x}_j) := \{\mathbf{x}_k - \mathbf{x}_j : ||\mathbf{x}_k - \mathbf{x}_j|| < r\}$.
- Output: $\mathbf{G}_{n\times d}$, where $\mathbf{G}_j = E(\mathfrak{N}(\mathbf{x}_j))$.
- WANT: the encoder E that receives a local point cloud and produces a vector.
- Implicit Assumption: Translation Invariance.

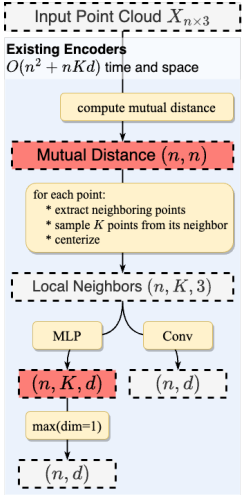


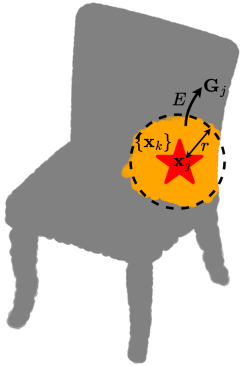
After Local Geometry Encoding...



Existing Methods

- Group and downsample.
- Memory Bottleneck: mutual distance matrix.
- Computation Bottleneck: require nK MLPs.

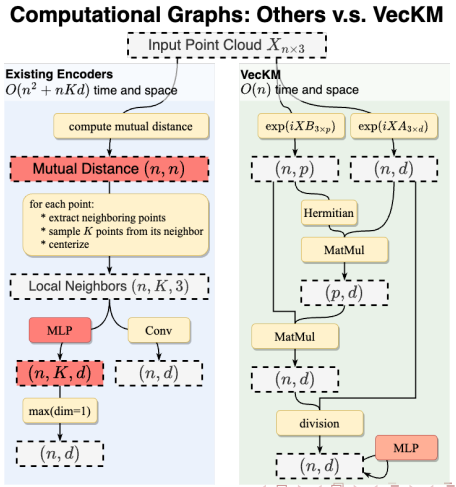




Our Solution: VecKM

A local geometry encoding that is

- **efficient to compute**.
- descriptive;
- robust to noise;
- simple to implement.



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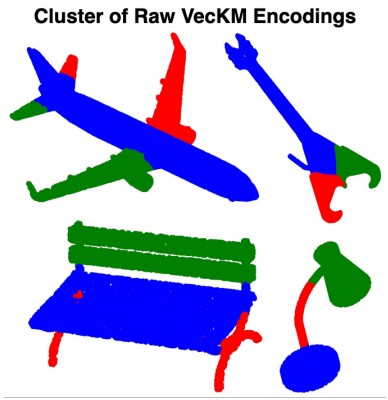

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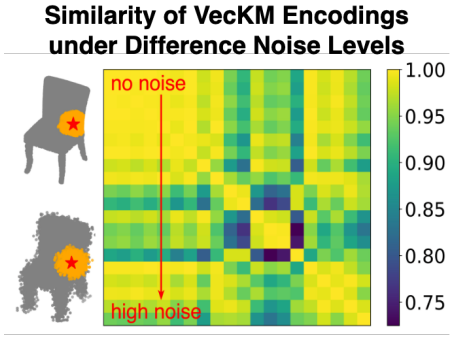

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A local geometry encoding that is

- efficient to compute.
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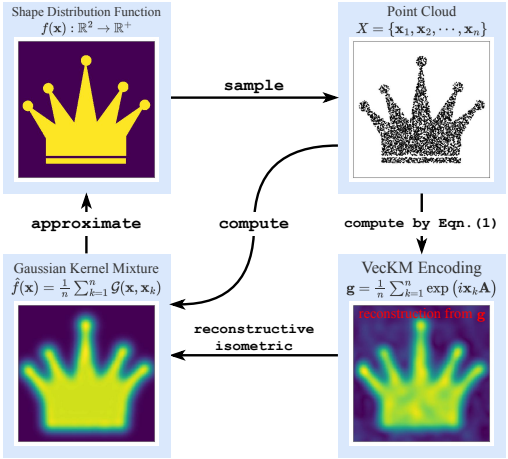
- **simple to implement**.

Initialization: $\mathbf{A}_{3\times d} \sim \mathcal{N}(0, \alpha^2)$, $\mathbf{B}_{3\times p} \sim \mathcal{N}(0, \beta^2)$.
Input: $\mathbf{X}_{n\times 3}$.

$$\mathcal{A}_{n\times d} = \exp\left(i\mathbf{X}_{n\times 3}\mathbf{A}_{3\times d}\right)$$
$$\mathcal{B}_{n\times p} = \exp\left(i\mathbf{X}_{n\times 3}\mathbf{B}_{3\times p}\right)$$
$$\mathbf{G}_{n\times d} = normalize\left(\left(\mathcal{B}\times\mathcal{B}^H\times\mathcal{A}\right)./\,\mathcal{A}\right)$$

Return: $MLP(\mathbf{G}_{n\times d})$.

Pointwise Local Geometry Encoding – Intuition



Key Observation:

$$\mathbf{g} = \frac{1}{n}\sum_{k=1}^{n} \exp(i\mathbf{x}_k\mathbf{A})$$

"**encodes**" the density distribution of $\{\mathbf{x}_k\}_{k=1}^{n}$.

Pointwise Local Geometry Encoding – Detail

Theorem (Pointwise Local Geometry Encoding)

Denote the neighbors of the point $\mathbf{x}_0$ *as* $\mathfrak{N}(\mathbf{x}_0) := \{\mathbf{x}_k - \mathbf{x}_0\}_{k=1}^n$. *The local geometry encoding of* $\mathbf{x}_0$ *is computed as*

$$\mathbf{g}_0 = \frac{1}{n}\sum_{k=1}^{n} \exp\left(i(\mathbf{x}_k - \mathbf{x}_0)\mathbf{A}_{3\times d}\right) \tag{1}$$

where i *is the imaginary unit and* $\mathbf{A} \in \mathbb{R}^{3\times d}$ *is a fixed random matrix where each element follows the normal distribution* $\mathcal{N}(0, \alpha^2)$.

Detailed proof in the paper. We skip here.

Dense Local Geometry Encoding

- Pointwise local geometry encoding:

$$\mathbf{g}_j = \frac{1}{n}\sum_{k=1}^{n} \exp\left(i(\mathbf{x}_k - \mathbf{x}_j)\mathbf{A}_{3\times d}\right) \cdot \mathbb{I}(||\mathbf{x}_k - \mathbf{x}_j|| < r)$$

- We can already compute the dense local geometry by grouping and computing one-by-one.
- However, no need!!! Because of factorizability:

$$\exp\left(i(\mathbf{x}_k - \mathbf{x}_j)\mathbf{A}_{3\times d}\right) = \exp(i\mathbf{x}_k\mathbf{A}_{3\times d})./\exp(i\mathbf{x}_j\mathbf{A}_{3\times d})$$

- By reusing computation, let $\mathbf{J}$ be the (sparse) adjacency matrix:

$$\mathcal{A}_{n\times d} = \exp(i\mathbf{X}_{n\times 3}\mathbf{A}_{3\times d})$$
$$\mathbf{G}_{n\times d} = (\mathbf{J}_{n\times n}\mathcal{A}_{n\times d})./\mathcal{A}_{n\times d} \quad (2)$$

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Dense Local Geometry Encoding – Version 1

$$\mathbf{J}[j,k] = \mathbb{I}(||\mathbf{x}_k - \mathbf{x}_j|| < r)$$
$$\mathcal{A}_{n\times d} = \exp(i\mathbf{X}_{n\times 3}\mathbf{A}_{3\times d})$$
$$\mathbf{G}_{n\times d} = (\mathbf{J}_{n\times n}\mathcal{A}_{n\times d})./\mathcal{A}_{n\times d}$$

Dense Local Geometry Encoding – Linear Complexity

- Recall Version 1:

$$\mathbf{J}[j,k] = \mathbb{I}(||\mathbf{x}_k - \mathbf{x}_j|| < r)$$
$$\mathcal{A}_{n\times d} = \exp(i\mathbf{X}_{n\times 3}\mathbf{A}_{3\times d})$$
$$\mathbf{G}_{n\times d} = (\mathbf{J}_{n\times n}\mathcal{A}_{n\times d})./\mathcal{A}_{n\times d}$$

- Trick: Replace $\mathbf{J}[j,k] = \exp(-\beta^2||\mathbf{x}_k - \mathbf{x}_j||^2)$.
- Motivation: $\mathbf{J} \approx [\mathcal{B}]_{n\times p}[\mathcal{B}^H]_{p\times n}$.
- Then

$$\mathbf{G}_{n\times d} = ([\mathcal{B}]_{n\times p}[\mathcal{B}^H]_{p\times n}\mathcal{A}_{n\times d})./\mathcal{A}_{n\times d} \tag{3}$$

Linear Time and Space!

Summary

Initialization: $\mathbf{A}_{3\times d} \sim \mathcal{N}(0, \alpha^2)$, $\mathbf{B}_{3\times p} \sim \mathcal{N}(0, \beta^2)$.
Input: $\mathbf{X}_{n\times 3}$.

$$\mathcal{A}_{n\times d} = \exp(i\mathbf{X}_{n\times 3}\mathbf{A}_{3\times d})$$
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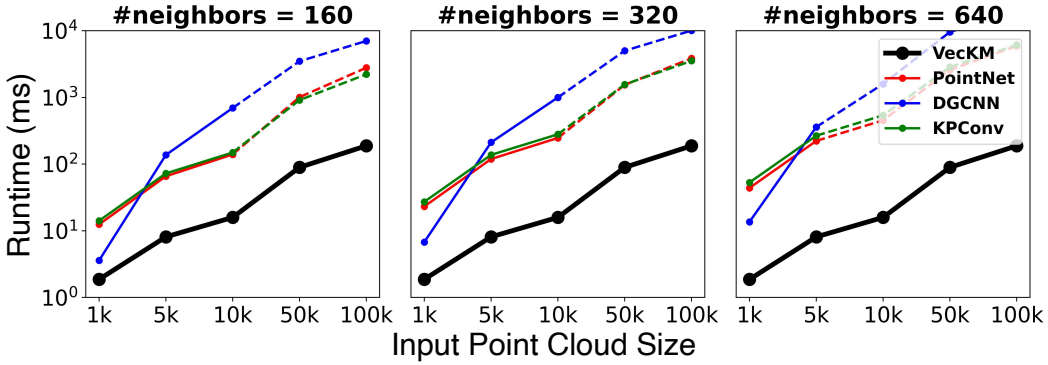
$$\mathbf{G}_{n\times d} = normalize\big((\mathcal{B} \times \mathcal{B}^H \times \mathcal{A}) ./ \mathcal{A}\big)$$

Return: $MLP(\mathbf{G}_{n\times d})$.

Runtime and Memory Cost



Normal Estimation on PCPNet Dataset

Table: Normal estimation RMSE on the PCPNet dataset.

	Perturbations				Density Variation		Average
	None	Low	Med	High	Gradient	Stripe	
KPConv, #kp=16	22.68	23.09	25.21	29.05	34.40	25.61	26.67
KPConv, #kp=32	22.74	22.21	24.08	28.25	32.24	24.94	25.74
KPConv, #kp=64	22.09	22.12	23.90	28.45	28.60	24.05	24.86
DGCNN, #nbr=32	24.08	24.04	25.19	28.24	27.12	27.55	26.03
DGCNN, #nbr=64	23.21	25.34	25.66	26.01	28.86	28.20	26.21
DGCNN, #nbr=128	18.46	18.71	20.38	25.62	23.01	21.29	21.24
PointNet, #nbr=300	14.98	16.30	20.19	26.83	23.68	19.00	20.17
PointNet, #nbr=500	16.10	16.54	21.38	26.93	26.06	18.89	20.99
PointNet, #nbr=700	15.59	16.25	20.99	26.21	24.66	17.87	20.27
VecKM (Ours)	**13.59**	**13.99**	**18.04**	**22.21**	**18.98**	**17.20**	**17.34**

Explanation of Robustness

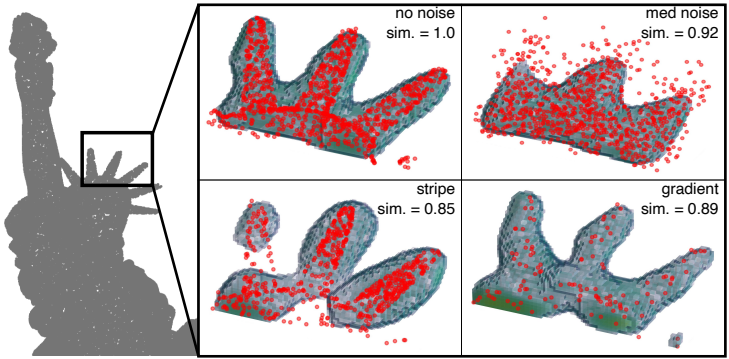


Figure: VecKM can reconstruct the local shape under corrupted inputs. The VecKM encodings remain highly similar under data corruptions.

Classification on ModelNet40

TABLE: Classification performance on the ModelNet40 dataset.

	Instance Accuracy	Avg. Class Accuracy	Inference Time (ms) (1 batch)	# parameters
PointMLP	93.1%	90.1%	325.85	13.2M
PointNet	90.8%	87.1%	3.04	1.61M
VecKM $\rightarrow$ PN	92.9%	89.7%	14.32	9.06M
Difference	$\uparrow$ 2.1%	$\uparrow$ 2.6%	not comparable	+7.61M
PointNet++	92.7%	89.4%	117.13	1.48M
VecKM $\leftrightharpoons$ PN++	93.0%	89.7%	65.78	3.94M
Difference	$\uparrow$ 0.3%	$\uparrow$ 0.3%	78% faster	+2.46M
PCT	92.9%	89.8%	149.72	2.88M
VecKM $\leftrightharpoons$ PCT	93.1%	90.6%	21.44	5.07M
Difference	$\uparrow$ 0.2%	$\uparrow$ 0.8%	5.98x faster	+2.19M

Part Segmentation on ShapeNet Dataset

Table: Part segmentation performance on the ShapeNet dataset.

	Instance mIoU	Avg. Class mIoU	Inference Time (ms) (1 batch)	parameters
PointMLP	85.1%	82.1%	240.39	16.76M
PointNet	83.1%	77.6%	15.1	8.34M
VecKM $\rightarrow$ PN	84.9%	81.8%	40.8	1.29M
Difference	$\uparrow$ 1.8%	$\uparrow$ 4.2%	not comparable	+7.05M
PointNet++	85.0%	81.9%	130.8	1.41M
VecKM $\leftrightharpoons$ PN++	85.3%	82.0%	65.9	1.50M
Difference	$\uparrow$ 0.3%	$\uparrow$ 0.1%	98% faster	+0.09M
PCT	85.7%	82.6%	145.2	1.63M
VecKM $\leftrightharpoons$ PCT	85.8%	82.6%	46.6	1.71M
Difference	$\uparrow$ 0.1%	0.0%	2.11x faster	+0.08M

Semantic Segmentation on S3DIS Dataset

Table: Semantic segmentation performance on the S3DIS dataset.

	Instance mIoU	Avg. Class mIoU	Overall Accuracy	Inference Time (ms) (per scene)	#parameters
PointNet++	64.05	71.52	87.92	96	0.968M
VecKM $\rightarrow$ PN++	67.48	73.53	89.33	391	1.11M
Difference	$\uparrow$ 3.43	$\uparrow$ 2.01	$\uparrow$ 1.41	not comparable	+0.142M
Point Transformer	69.29	75.66	90.36	559	7.77M
VecKM $\leftrightharpoons$ PT	69.53	75.84	90.39	447	7.93M
Difference	$\uparrow$ 0.24	$\uparrow$ 0.18	$\uparrow$ 0.03	20% faster	+0.16M

Real-Time Optical/Normal Flow Estimation from Event Cameras with VecKM

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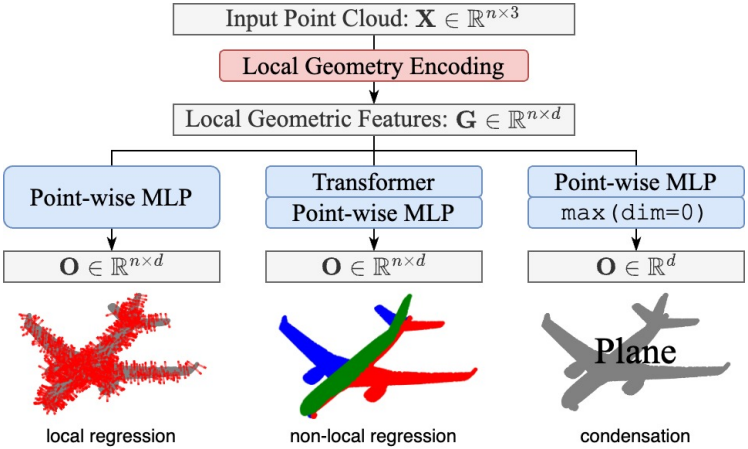
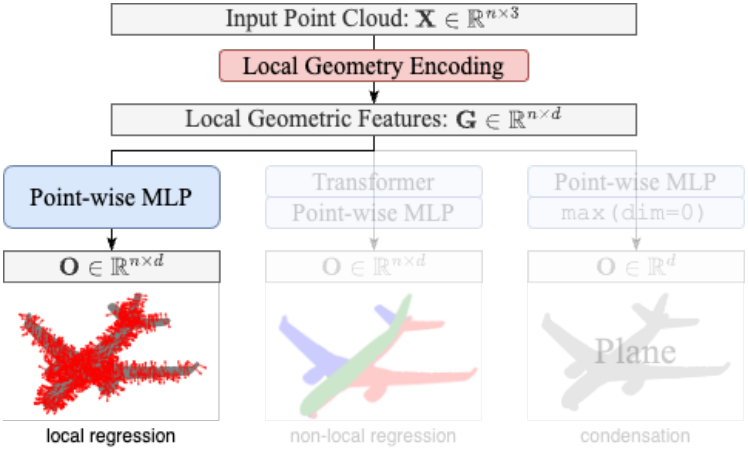
Recall: After Local Geometry Encoding...



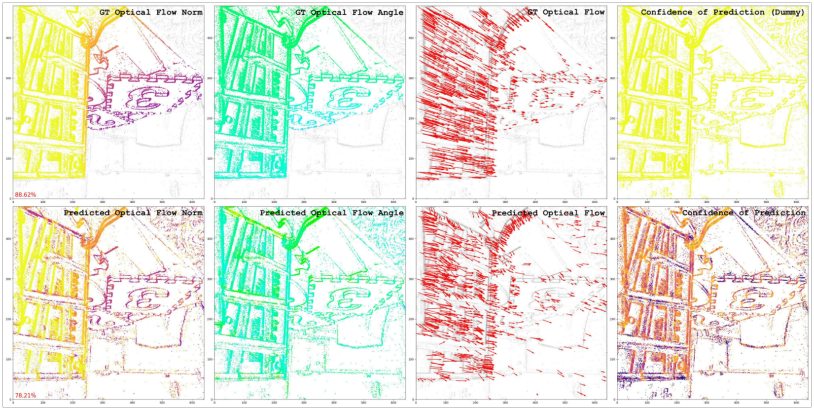
We focus on this:



Simply train an optical flow estimator supervisedly?

It turns out to be very good!

Predicting per-event flow within 20 ms (∼50k events) requires 20 ms. So 40 ms latency.



Current Challenge on GPU

- Have to accumulate events and estimate flows in batches.
- Due to inplace summation must be blocking.

Input: $\mathbf{X}_{n\times 3}$.

$$\mathcal{A}_{n\times d} = \exp(i\mathbf{X}_{n\times 3}\mathbf{A}_{3\times d})$$
$$\mathcal{B}_{n\times p} = \exp(i\mathbf{X}_{n\times 3}\mathbf{B}_{3\times p})$$
$$\mathbf{G}_{n\times d} = normalize\big((\mathcal{B}\times\mathcal{B}^H\times\mathcal{A})\,./\,\mathcal{A}\big)$$

Input: single event $\mathbf{x}_{1\times 3}$.

$$\alpha_{1\times d} = \exp(i\mathbf{x}_{1\times 3}\mathbf{A}_{3\times d})$$
$$\beta_{1\times p} = \exp(i\mathbf{x}_{1\times 3}\mathbf{B}_{3\times p})$$
$$[B^HA]_{p\times d} \mathrel{+}= \beta^H\times\alpha$$
$$\mathbf{G}_{n\times d} = normalize\big((\beta_{1\times p}\times[B^HA]_{p\times d})\,./\,\alpha_{1\times d}\big)$$

Maybe on Neuromorphic Hardware?

The inplace summation is non-blocking.