Comments on Chaewoon's Paper

1 Title and Introduction

1. The title does not mention that you are looking at proofs that use Ramsey Theory. Also, the title indicates that you will prove that LOTS of domains have an infinite number of primes, whereas what you ard doing is BOTH applying to other domains AND seeing where the proofs fall apart in other domains.

   Here is a better title

   Examining Proving Primes Infinite Using Ramsey Theory

2. First sentence should be more direct:

   The first proof that the primes are infinite is attributed to Euclid and is dated at (approximately) 300 BC.

3. You seem to imply that $\mathsf{Z}[i]$ has a finite number of primes which is not true. Better off not being to specific about domains: Will suggest a change later.

4. You don't mention that you will focus on proofs that use Ramsey Theory until the end of the abstract. I will suggest a change later.

Here is the change I suggest:

The first proof that the primes are infinite is attributed to Euclid and is dated at (approximately) 300 BC. Since then there have been hundreds of proofs that the primes are infinite. We examine several that use Ramsey Theory.

For each of the proofs we consider we do the following: (1) Extend the proof to work in other integral domains. There will be a condition on the domain to make it work. (2) See how the conditions fail when applied to a domain that has a finite number of primes. (3) See which domains other than Z *the proof succeeds on.*

2 Definitions

1. *You should define a ring.*

2. *You should define* Z_n, N, Z, Q,$\mathsf{Z}[i]$, R. *directly and not just in passing.*

3. Q_2 *is not standard. You need to define it.*

4. *You have However, natural numbers* N *is not an integral domain because it does not qualify as a ring.*

   Better:

   However, N *is not an integral domain because it does not have additive inverses, so its not even a ring.*

5. *You have Some integers modulo* n Z_n *are not integral domains either.*

   This does not parse well. And you later use the term zero divisor which you have not defined. Fortunately you have defined Z_n *already so you can say:*

   For n *composite,* Z_n *is not an integral domain. For example, in* Z_6, $2 \times 3 = 0$ *but* $2 \neq 0$ *and* $3 \neq 0$. *For* n *prime,* Z_n *is known to be an integral domain.*

6. *Irreducibles. Your first sentence is to long and has an IF without a THEN*

   Better:

   Let p *be a nonzero, nonunit, element of* D. *If whenever* $p = ab$, *either* a *or* b *is a unit, then* p *is irreducible.*

7. *General-good if the word you are defining is in italics or some font to make it stand out, as I did in the last sentence.*

8. *You should have Primes as a separate boldface definition and then remark that all irreducibles are primes but not conversely.*

9. *Rather than say for our purposes in this paper primes are irreducibles you should say for all of the domains in this paper, primes and irreducibles are the same*

   (I may look at domains where they are NOT the same and see what happens with these Ramsey-Primes proofs, but not today.)

10. *Composites. exists non-unit* $a, b \in \mathsf{D} - \{0\}$ *where* $n = ab$. *should be non-units, plural.*

11. *(Infinite) Unique Factorization Domains. Just curious- are all of the domains we deal with unique factorization domains? If so then say so.*

    (I may look at domains that are NOT unique factorization and see what happens with these Ramsey-Primes proofs, but not today.)

12. *Number of Primes. A bit odd to ask if* Z *has twice as many primes a* N *since both sets are countable. Better to say When looking at the primes in* Z *do we count both 2 and* -2*?*

13. *The way we count primes- using equiv classes- is very important. It should stand out more. In LaTeX I would do*

    Convention 2.1 The phrase D *has an infinite number of primes* will really mean D *has an infinite number of equivalence classes of primes with the equivalence* $a \equiv b$ *if there exists* $u \in U$ *with* $au = b$.

    Find some way to make this convention stand out.

14. *You correctly characterize the primes in* $\mathsf{Z}[i]$. *It is unclear to the reader if this is supposed to be obvious. You can write The following is known: the primes in* $\mathsf{Z}[i]$ *are ...*

3 Notation and Lemmas

1. $v_2(36) = 4$ *means* 2^4 *divides 36 which is not true.*

2. *You say that we define* $v_p(0) = \infty$ *for the sake of convenience. Better to say that we define it so that the rules 1,2,3 still hold for 0.*

3. *Fermat's last theorem. You have*

   For all $n \geq 3 \in \mathsf{Z}$,

   This looks odd since you are saying $3 \in \mathsf{Z}$ *and not* $n \in \mathsf{Z}$.

   Better:

   For all $n \in \mathsf{Z}$, $n \geq 3$,

4. *The Fundamental Theorem of Arithmetic. I don't think you need the parenthesis around* $p^{v_p(n)}$.

5. *Schur's Theorem. General rule- IF statements should have THEN's. If its awkward to do that then don't use an IF statement. For Schur's theorem can do*

   For all c, for all $\chi \mathsf{N} \rightarrow [c]$*, there exists* $x, y, z \in \mathsf{N} - \{0\}$ *such that* $x + y = z$ *and* x, y, z *are the same color.*

6. *VDW- similar.*

7. *You go from VDW to GEN POLY VDW. You should include in between them Poly VDW. And avoid the IF without a THEN as above.*

8. *You say you will show* N *has infinitely many primes. Should you do* Z *instead since it is an integral domain?*

9. *You say you will do 5 proofs-you should point out that you will do the original proof and 4 proofs that use Ramsey Theory.*

4 Euclid's Proof

1. *In an increasing order should be In increasing order This occurs elsewhere as well.*

2. *Now consider a number* $n = p_1 \cdots p_k + 1$*.*

   Should be

   Now consider the number $n = p_1 \cdots p_k + 1$*.*

3. *You mention norms. You did not define norms.*

4. *I just noticed this: your End-of-Proof markers do not stand out. Make it a filled-in-box instead of a hollow box, or make it something that stands out.*

5. *The paragraph The following four proofs ... is in the section titled Euclid's Proof. Even though it would be short, that paragraph can be in a section.*

5 EG Proof

1. *Second line you have Consider a number $n \in \mathsf{D}$. There is no D in this proof. I think you mean N. The symbol D appears later in the proof, and in some of the other proofs.*

2. *Before saying Consider a number $n \in \mathsf{D}$ (which you will correct to N) you should say*

   We will define a coloring of N and later apply Schur's Theorem to it.

   This comment applies to all of your Ramsey-prime proofs.

3. *After the proof you write ... when this proof is applied to other integral domains, the application of FLT on the domain must be validated in order to successfully apply the proof.*

   This is not true. You need to look at the VARIANT of FLT that you allude to later, allowing units as coefficients.

   In fact, FLT DOES hold for the rationals:

   $$(a/b)^n + (c/d)^n = (e/f)^n$$

   Multiply by $(bdf)^n$ to get

   $$(adf)^n + (cbf)^n = (ebd)^n$$

   which violates the usual FLT.

4. *The sentence*

   In other words $R(a) = R(a+d) = R(a+2d) = R(a+3d)$

   goes off the end of the line in a way that makes the sentence hard to parse. better to either center the equation on a line or just push it to all be on one line.

5. *After the proof you write ... when this proof is applied to other integral domains, the application of Fermat's Four-square Theorem on the domain must be validated in order to successfully apply the proof.*

   Again, not true. You need to modify the 4-square theorem to involve units, as you do later.

6. *After the paper is corrected I will ask around about* $\mathsf{Z}[i]$ *and the modified version of the 4-square theorem. Also, you should find a proof of the 4-square theorem and see if you can modify the proof to obtain the version of it we need for* $\mathsf{Z}[i]$.

6 Apologe's Proof

1. *You should say that the number of colors is* $2^{2k} = 4^k$ *since its easy to see that its* 2^{2k}. *If you just say* 4^k *people might look for* k *slots that each take one of 4 values.*

2. *Now consider all* $p \in \{p_1, p_2, \ldots, p_k\}$

   The sentence goes over the edge badly. Fix this somehow.

3. *Actually there are NOT* 2^{2k} *colors.*

   If $s_i = 0$ *then* $r_i = 0$ *so some of the colors you allow can't happen. This is not important but you should mention it.*

4. *The proof should be split into paragraphs better. The proof that* $v_p(a) > v_p(d)$ *leads to a contradiction should be one paragraph. Then the proof that* $v_p(a) = v_p(d)$ *should be another sep paragraph. Then the* $v_p(a) < v_p(d)$ *sentence and the end of the proof should be a third paragraph.*

5. *Lots of the equations should be centered for readability.*

6. *The usual comment: for other domains do you need FTA or some variant of it that uses units?*

7 GOS Proof

1. *The statement of the theorem should be clearer.*

   Here is better:

   Let D *be an infinite unique factorization domain with a finite number of units. The DPVDW implies that* D *has infinitely many primes.*

2. *Not an objection but a question: In the other proofs you did the proof for* N *and then commented on how to extend it to other domains, and*

also why the proof did not work for Q *or* Q_2*. Here you prove the general version directly. Why?*