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Is there an m such that they **cannot** intersect in two places?
Next Slide

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So some number $\geq 2^3 = 8$ divides α. But $\alpha \in \{1, 2, 3, 4\}$.
End of Proof

An Easy Number Theory Lemma

Lemma Let $k \geq 3$. $(\exists m = m(k))$ such that:
For all $\alpha, \beta \in \{1, \ldots, k\}$ there is **no** (d_1, d_2) with $d_1 \neq d_2$ such that

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$$\alpha p_1^{a_1 m} \cdots p_\ell^{a_\ell m} = \beta q_1^{b_1 m} \cdots q_{\ell'}^{b_{\ell'} m}$$

Let r be a prime that divides α. Since α, β are rel prime r does not divide β. Hence r is some q_i. Since there are no other q_i's on the LHS, $q_i^{b_i m}$ must divide α. The smallest this can be is 2^m. Hence take m such that $2^m > k$ for a contradiction.

End of Pf

A Theorem about Intersecting APs

Thm Let $k \geq 3$ and $m = m(k)$. If A_1 is a k-AP with diff d_1^m and A_2 is a k-AP with diff d_2^m, with $d_1 \neq d_2$, then $|A_1 \cap A_2| \leq 1$.
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So we have $\alpha, \beta \in [k-1]$ with

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This contradicts the definiton of $m = m(k)$.
End of Pf