BILL, RECORD LECTURE!!!!

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All 2-Coloring Of the Plane have a Red 2-Stick or Blue 3-stick

Exposition by William Gasarch-U of MD

Credit Where Credit is Due

The main result in these slides is due to Szlam (1999).

Chromatic Number of the Plane: Review

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Answer on next slide

For All 2-Colorings of The Plane . . .

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Case 2 There is no **Blue** ℓ_3.

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This is well defined because of the case we are in.

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So $(x_1, y_1 + 1)$ and $(x_2, y_2 + 1)$ are a **Red** ℓ_2. Done!

Can Prove Result About (ℓ_2, ℓ_4)

Using that the Chromatic Number of the Plane (χ) is ≤ 4 one can easily prove the following:

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Juhasz prove the above theorem without using $\chi \leq 4$. In fact, he prove the theorem in 1979, 39 years before $\chi \leq 4$ was proven.

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We will soon present statements about $\mathbb{R}^2 \to (\ell_2, \ell_n)$

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$\mathbb{R}^2 \to (\ell_2, \ell_n)$?

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Positive Results		
Szlam, 1999	$(\forall X)[\lvert X\rvert = 3]$ $\mathbb{R}^2 \to (\ell_2, X)$	$\mathbb{R}^2 \to (\ell_2, \ell_3)$
Juhasz, 1979	$(\forall X)[\lvert X\rvert = 4]$ $\mathbb{R}^2 \to (\ell_2, X)$ Congruence. See Paper.	$\mathbb{R}^2 \to (\ell_2, \ell_4)$
Tsaturian, 2017	$\mathbb{R}^2 \to (\ell_2, \ell_5)$	$\mathbb{R}^2 \to (\ell_2, \ell_5)$
Arman-Tsaturian, 2017	$\mathbb{R}^3 \to (\ell_2, \ell_6)$	NONE
Negative Results		
Csizmadia-Togh 1994	$\exists X \subseteq \mathbb{R}^2$, $\lvert X\rvert = 8$ $\mathbb{R}^2 \not\to (\ell_2, X)$	NONE
Conlon-Fox, 2018	$(\forall n \ge 2)$ $\mathbb{R}^n \not\to (\ell_2, \ell_{10^{25}})$	$\mathbb{R}^2 \not\to (\ell_2, \ell_{10^{25}})$

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Open Narrow the gap between $\mathbb{R}^2 \rightarrow (\ell_2, \ell_5)$ and $\mathbb{R}_2 \not\rightarrow (\ell_2, \ell_{10^{25}})$.

$\mathbb{R}^2 \to (\ell_3, \ell_n)$?

All of the negative results hold for all $n \geq 2$. Our interest is in the $n = 2$ case.

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Conlon-Wu, 2022	$\mathbb{R}^n \not\to (\ell_3, \ell_{10^{50}})$
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The results of Fuhrer-Togh and Currie-Moore-Yip are messy. For some reasonable values of n find nice proofs that $\mathbb{R}^2 \not\to (\ell_3, \ell_n)$.

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Find $a, b \geq 5$ such that $\mathbb{R}^2 \to (\ell_5, \ell_a)$, $\mathbb{R}^2 \not\to (\ell_5, \ell_b)$. It is possible that a does not exist and $b = 5$.

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Some of these problems may be independent of ZF

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Fox's answer surprised me:

Some of these problems may be independent of ZF

My Opinion This would be both awesome and awful.

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