BILL START RECORDING

HW 10 Solutions

Problem 1 Set Up

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$\mathrm{VC} = \{(G, k) \colon G$ has a VC of size $\leq k\}$.

It is known that VC is NP-complete.

Problem 1a

Want: a connected graph on 1000 vertices that has a VC of size 1.

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$U = \{1\}$ is a VC of size 1.

Problem 1b

Want: a connected graph on 1000 vertices so that the smallest vertex cover for it has size 999.

Take the complete graph on 1000 vertices.
$V = \{1, \ldots, 1000\}$
$E = \{(i, j) \colon 1 \leq i < j \leq 1000\}$
$U = \{1, 2, \ldots, 999\}$ is a VC of size 999.
We leave it to the reader that there is not a smaller VC.

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$U = \{2, 4, \ldots, 1000\}$ is a VC of size 500.

No set of size 499 works. Left to the reader.

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We sketch two algorithms.

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4) Repeat on each side until depth 1000.

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Much better in practice, and has been improved.

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Takes time $O(n + m) + k^{k^2} \leq O(n) + 1000^{1000^2}$.

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The Best Known Algorithm takes time $O(kn + 1.2738^k)$.

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It works very well in practice.

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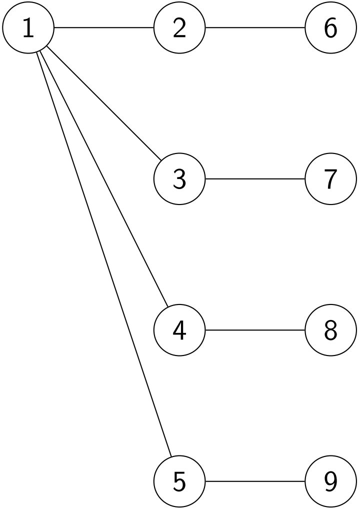

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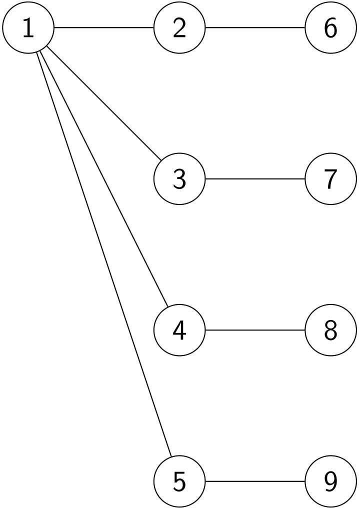

Greedy algorithm produces $\{1, 2, 3, 4, 5\}$, 5 vertices.

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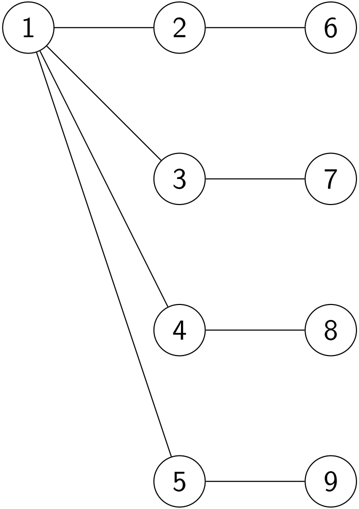

Greedy algorithm produces $\{1, 2, 3, 4, 5\}$, 5 vertices.
Optimal is $\{2, 3, 4, 5\}$, 4 vertices.

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It is known that DS is NP-complete.

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For all vertices there are no neighbors, so every vertex is in the DS.

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Dom Set: $\{1, 3, \ldots, 999\}$.

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The following people emailed me a solution:
Aiden Paul
Alex Mendelson (TA)
Anish Bhupalam
Ethan Price
Roc Yu

Problem 2c: Making it Connected

I originally asked for a **connected** graph on 1000 vertices that has a DS of size 500 but not 499.

I thought C_{1000} would work but Nolawe-Isaac-Felix told me it does not.

I didn't intend for this to be a hard problem so I removed **connected** but also asked the class to, if they DID find a connected graph, email it to me for mine and the classes enlightenment.

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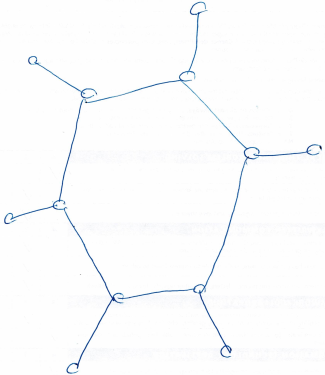
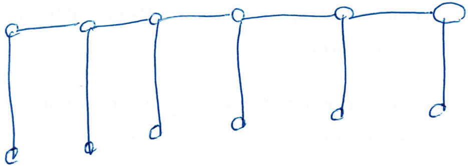
The next two slides show graphs on 12 vertices that have a DS of size 6 but not 5. They convey the ideas.

First Graph on 12 Vertics, DS Size 6, Not 5



Second Graph on 12 Vertics, DS Size 6, Not 5



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The question is UNKNOWN TO SCIENCE!

Problem 2f

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