BILL START RECORDING

HW07 Solutions

Problem 1a

Problem 1a

$$S \rightarrow AABB$$
$$S \rightarrow ABAB$$
$$A \rightarrow AAA$$
$$B \rightarrow BBB$$
$$A \rightarrow a$$
$$B \rightarrow b.$$

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Give ∞ number of strings NOT in $L(G)$.

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So the set $\{w \colon |w| \equiv 1 \pmod 2\}$ works.

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Give ∞ number of strings NOT in $L(G)$. Several Answers.

So the set $\{w : |w| \equiv 1 \pmod 2\}$ works.

a^* (shout out to Issac-Felix-Nolawe!). Also b^*

Problem 1b

Problem 1b

$$S \to AABB$$
$$S \to ABAB$$
$$A \to AAA$$
$$B \to BBB$$
$$A \to a$$
$$B \to b.$$

Problem 1b

$S \to AABB$
$S \to ABAB$
$A \to AAA$
$B \to BBB$
$A \to a$
$B \to b.$

Convert to Chomsky Normal Form.

Problem 1b

$$S \rightarrow AABB$$
$$S \rightarrow ABAB$$
$$A \rightarrow AAA$$
$$B \rightarrow BBB$$
$$A \rightarrow a$$
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Convert to Chomsky Normal Form.

We do two of the rules and leave the rest to the reader.

Problem 1b

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BECOMES the following set of rules

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$S \to A[ABB]$.

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Note Some of you might have done

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$[ABB] \rightarrow [A][BB]$

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$[ABB] \to [A][BB]$

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BECOMES the following set of rules

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Note Some of you might have done

$[ABB] \rightarrow [A][BB]$

$[A] \rightarrow A$

We allowed this even though its not quite right.

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$A \to AAA$
BECOMES the following
$A \to A[AA]$
$[AA] \to AA$

Problem 2a

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Give an algorithm that will, GIVEN a DFA M produce a CFG G such that $L(M) = L(G)$.

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Given $M = (Q, \Sigma, s, F)$ create the following CFG.

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Rules

For every δ-transition $\delta(q, \sigma) = p$ we have the rule $q \rightarrow \sigma p$.

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Give an algorithm that will, GIVEN a DFA M produce a CFG G such that $L(M) = L(G)$.
Given $M = (Q, \Sigma, s, F)$ create the following CFG.

Nonterminals: Q

Start NT: s

Rules

For every δ-transition $\delta(q, \sigma) = p$ we have the rule $q \rightarrow \sigma p$.

For every $f \in F$ we have the rule $f \rightarrow e$.

Problem 2b

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$$L = \{w \colon \#_a(w) \equiv 0 \pmod 2 \wedge \#_b(w) \equiv 0 \pmod 2\}.$$

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We form the CFG using the DFA

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For $0 \leq i, j \leq 1$:

$(i,j) \to a(i+1 \pmod 2, j)$

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$(0,0) \rightarrow e$.

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$$L_n = \{w \colon \#_a(w) \equiv 0 \pmod{n} \wedge \#_b(w) \equiv 0 \pmod{n}\}.$$

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$Q = \{(i, j) : 0 \leq i, j \leq n - 1\}$.

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We form the CFG using the DFA
$N = \{(i,j) \colon 0 \leq i,j \leq n-1\}$. $S = (0,0)$.

$(i,j) \to a(i+1 \pmod{n}, j)$

$(i,j) \to b(i, j+1 \pmod{n})$

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$N = \{(i,j)\colon 0 \leq i,j \leq n-1\}$. $S = (0,0)$.

$(i,j) \to a(i+1 \pmod{n}, j)$

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$(0,0) \to e$

Number of rules: $2n^2$.

Problem 3

Problem 3

$E_0 \to a \mid e$ (so $L(E_0) = \{e, a\}$.
$E_1 \to E_0 E_0 \mid e$ (so $\{e, a^2\} \subseteq L(E_1)$).
$E_2 \to E_1 E_1 \mid e$ (so $\{e, a^4\} \subseteq L(E_2)$).
$E_3 \to E_2 E_2 \mid e$ (so $\{e, a^8\} \subseteq L(E_3)$).
$E_4 \to E_3 E_3 \mid e$ (so $\{e, a^{16}\} \subseteq L(E_4)$).
$E_5 \to E_4 E_4 \mid e$ (so $\{e, a^{32}\} \subseteq L(E_5)$).
$E_6 \to E_5 E_5 \mid e$ (so $\{e, a^{64}\} \subseteq L(E_6)$).

Problem 3

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$E_1 \to E_0 E_0 \mid e$ (so $\{e, a^2\} \subseteq L(E_1)$).
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$E_6 \to E_5 E_5 \mid e$ (so $\{e, a^{64}\} \subseteq L(E_6)$).
$S \to E_0 E_1 E_2 E_3 E_4 E_5 E_6$

Problem 3a

Show that $L(G) = \{e, a, a^2, \ldots, a^{127}\}$ (Hint: Use that every number can be written in base 2.)

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There exists $0 \leq x_1, \ldots, x_6 \leq 1$.

Problem 3a

Show that $L(G) = \{e, a, a^2, \ldots, a^{127}\}$ (Hint: Use that every number can be written in base 2.)
$0 \leq x \leq 127$.
There exists $0 \leq x_1, \ldots, x_6 \leq 1$.

$$x = x_6 \times 2^6 + x_5 \times 2^5 + x_4 \times 2^4 + x_3 \times 2^5 + x_2 \times 2^2 + x_1 \times 2^1 + x_0 \times 2^0.$$

$S \to E_0 E_1 E_2 E_3 E_4 E_5 E_6$
If $x_i = 0$ then do $E_i \to e$
If $x_i = 1$ then do $E_i \Rightarrow a^{2^i}$.

Problem 3b

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Convert the grammar to CNF.

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Problem 3b

Convert the grammar to CNF.
$S \rightarrow E_0 E_1 E_2 E_3 E_4 E_5 E_6$
BECOMES

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$S \rightarrow E_0 E_1 E_2 E_3 E_4 E_5 E_6$

BECOMES

$S \rightarrow E_0 [E_1 E_2 E_3 E_4 E_5 E_6]$

Problem 3b

Convert the grammar to CNF.

$S \to E_0 E_1 E_2 E_3 E_4 E_5 E_6$

BECOMES

$S \to E_0[E_1 E_2 E_3 E_4 E_5 E_6]$

$[E_1 E_2 E_3 E_4 E_5 E_6] \to E_1[E_2 E_3 E_4 E_5 E_6]$

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Convert the grammar to CNF.

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$[E_1 E_2 E_3 E_4 E_5 E_6] \to E_1 [E_2 E_3 E_4 E_5 E_6]$

$[E_2 E_3 E_4 E_5 E_6] \to E_2 [E_3 E_4 E_5 E_6]$

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$[E_3 E_4 E_5 E_6] \to E_3 [E_4 E_5 E_6]$

Problem 3b

Convert the grammar to CNF.

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$[E_1 E_2 E_3 E_4 E_5 E_6] \rightarrow E_1 [E_2 E_3 E_4 E_5 E_6]$

$[E_2 E_3 E_4 E_5 E_6] \rightarrow E_2 [E_3 E_4 E_5 E_6]$

$[E_3 E_4 E_5 E_6] \rightarrow E_3 [E_4 E_5 E_6]$

$[E_4 E_5 E_6] \rightarrow E_4 [E_5 E_6]$

Problem 3b

Convert the grammar to CNF.

$S \rightarrow E_0 E_1 E_2 E_3 E_4 E_5 E_6$

BECOMES

$S \rightarrow E_0[E_1 E_2 E_3 E_4 E_5 E_6]$

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Problem 3c

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So there are $2t$ NT's. So FILLIN is $2t$.

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Hence

For all n there exists a CNFG G that generates $\{e, a, a^2, \ldots, a^n\}$ that has $O(\log n)$ NTs.