BILL START RECORDING

HW06 Solutions

Problem 1a, 1b

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$\{w\colon \#_a(w) \equiv 0 \pmod 3\}$.

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We show grammars for one of the 9 sets. The rest are either similar or very easy.

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Note that $|\alpha| = |\alpha_1| + |\alpha_2| + 3$.

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See next slide for exciting finish!

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$3A \leq B + 1$

Take $A = 1$ and $B = 3$. SO $f(n) = n + 3.$

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FINE to use $f(n) = O(n)$ or similar.

Notice that the CFG is in Chomsky NF even though we did not require that.

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On HW07 we show there is a way to go from a DFA to a CFG in CNF with linear blowup so YES, we can do better.