BILL START RECORDING

HW04 Solutions

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Discuss how you would do this problem with your table mates.

Problem 2

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Want UPPER BOUND on numb of states in DFA-classifier-mod-n.

Problem 2 (cont)

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n states: $(0,0)$, $(1,0)$, ..., $(n-1,0)$. Weight a_1 takes you to;

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n states $(0, L-2)$, $(1, L-2)$, …, $(n-1, L-21)$. Weight a_{L-1} takes you to:

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n states: $(0, 0)$, $(1, 0)$, …, $(n-1, 0)$. Weight a_1 takes you to;

n states: $(0, 1)$, $(1, 1)$, …, $(n-1, 1)$. Weight a_2 takes you to:

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n states $(0, L-2)$, $(1, L-2)$, …, $(n-1, L-21)$. Weight a_{L-1} takes you to:

n states labelled $(0, 0, g)$, $(0, 1, g)$, …, $(0, n-1, g)$.

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We are then in a grid-machine with states $\{(i,j,g) \colon 0 \le i \le n-1 \text{ and } 0 \le j \le m-1\}$.

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Number of states is $1 + (L-1)n + nm$.

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Give a table for a DFA classifier mod n.

The Mod and the Pattern

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$\delta((i, 3), \sigma) = (i + 144\sigma \pmod{224}, 0, g)$.

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$\delta((i,1,g),\sigma) = (i + 64\sigma \pmod{224}, 2, g)$.
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$\delta((i, 4, g), \sigma) = (i + 160\sigma \pmod{224}, 5, g)$.

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$\delta((i, 5, g), \sigma) = (i + 32\sigma \pmod{224}, 0, g)$.