BILL, RECORD LECTURE!!!!

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Deterministic Finite Automata (DFA)

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Notation $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \cdots$. is the set of all strings including e.

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Let $x, y \in \Sigma^*$. Then xy is the **concatenation of x and y**. We sometimes write it as $x \cdot y$.

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For example, aab is a prefix of $aabbaaba$.

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- $25 \equiv 35 \pmod{10}$.
- $100 \equiv 2 \pmod{7}$ since $100 = 7 \times 14 + 2$.

Modular Arithmetic II: Convention

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When dealing with mod n we assume the entire universe is $\{0, 1, \ldots, n-1\}$.

Modular Arithmetic: $+, -, \times$

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4. Division: Next Slide.

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No such x exists.

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No such x exists.
Fact: A number y has an inverse mod 26 if y and 26 have no common factors. Numbers that have an inverse mod 26:

$$\{1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25\}$$

Examples of DFA's Before Formal Def

We do examples of DFA's before defining them formally.

$$\{w : \#_a(w) \equiv 1 \pmod{2} \wedge \#_b(w) \equiv 2 \pmod{3}\}$$

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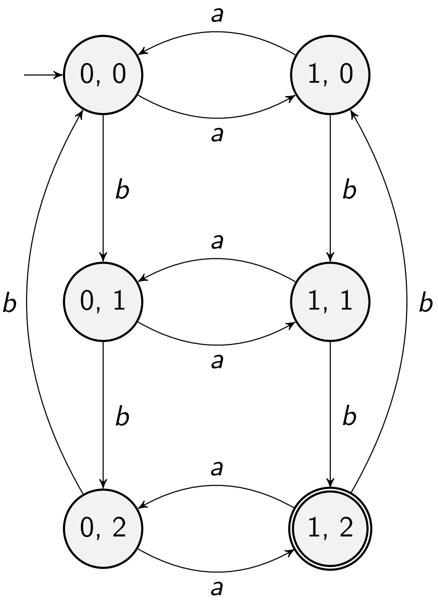

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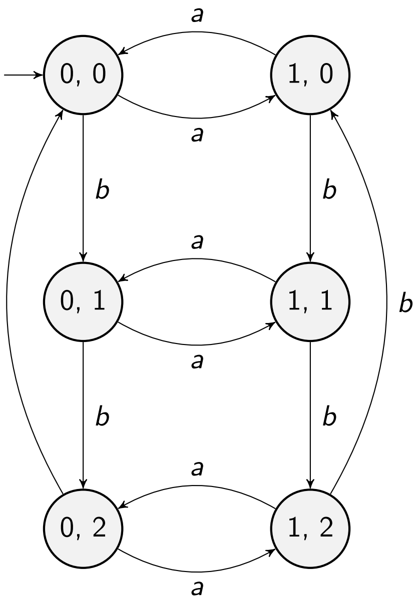

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The first DFA **accepted** $(1, 2)$-strings and **rejected** the rest.

The second DFA **classifies** strings without judgment.

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Proof Assume DFA M has ≤ 5 states.

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$\{w : \#_a(w) \equiv 1 \pmod{2} \wedge \#_b(w) \equiv 2 \pmod{3}\}$

Thm Any DFA for the lang has at least 6 states.
Proof Assume DFA M has ≤ 5 states.
On input e, the empty string, goes to state q_e.
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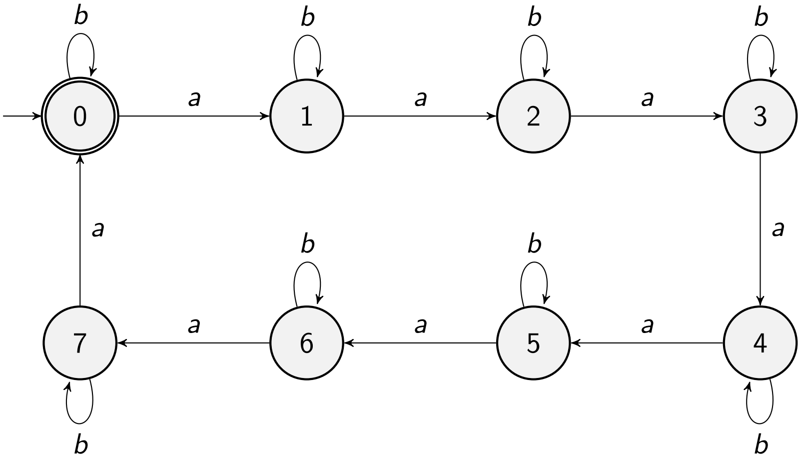
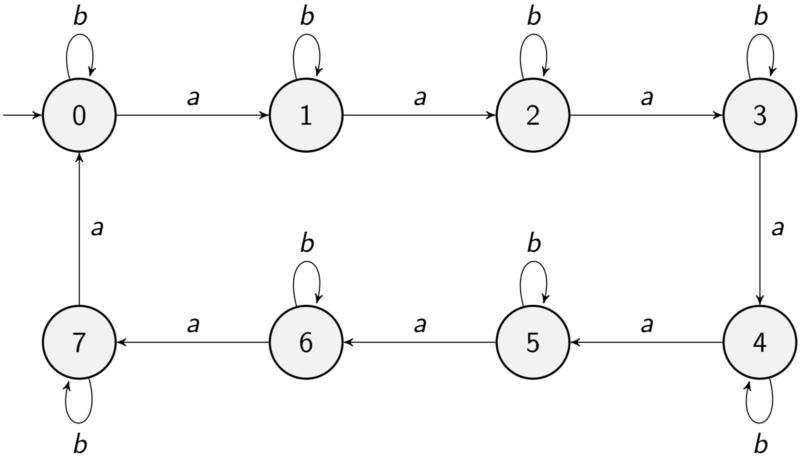
Contradiction.
Would need to do this argument with all pairs OR do it in a more general way. Might be on a HW, MIDTERM, or FINAL.

$\{w : \#_a(w) \equiv 0 \pmod 8\}$



DFA-Classifier for $\{w : \#_a(w) \equiv 0 \pmod 8\}$



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Might be on a HW or exam.

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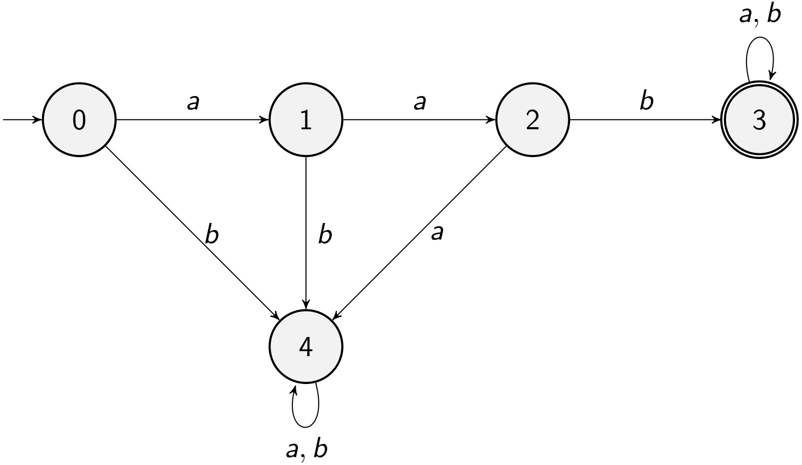

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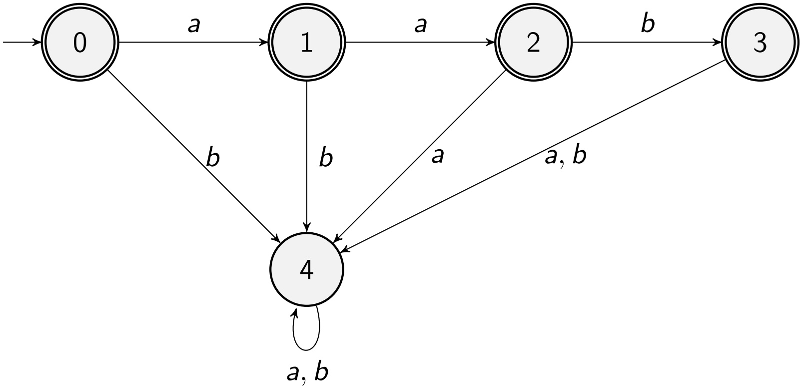
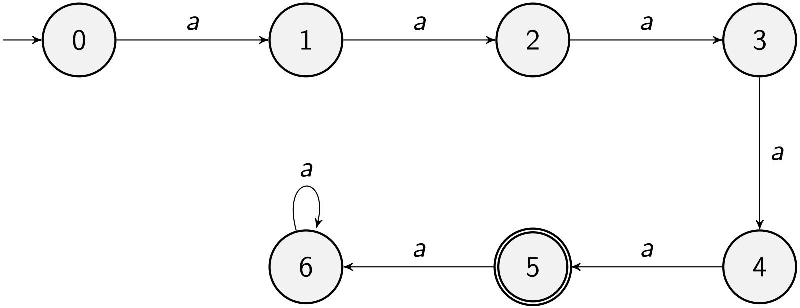


Example of DFA: $\{aaaaa\}$



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The DFA we drew for this had 7 states.
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Two of q_i, q_j are the same state. See next slide.

Continuing proof

Assume $i < j$ and $q_i = q_j = q$.
Note that $i \leq 5$.
Input a^i ends in state q_i.
Input a^j ends in state q_j.
$a^i a^{5-i} = a^5$ ends in ACCEPT state.
$a^j a^{5-i} = a^{5+j-i}$ ends in REJECT state since $5 + j - i > 5$.
But these strings end in SAME state, so contradiction.

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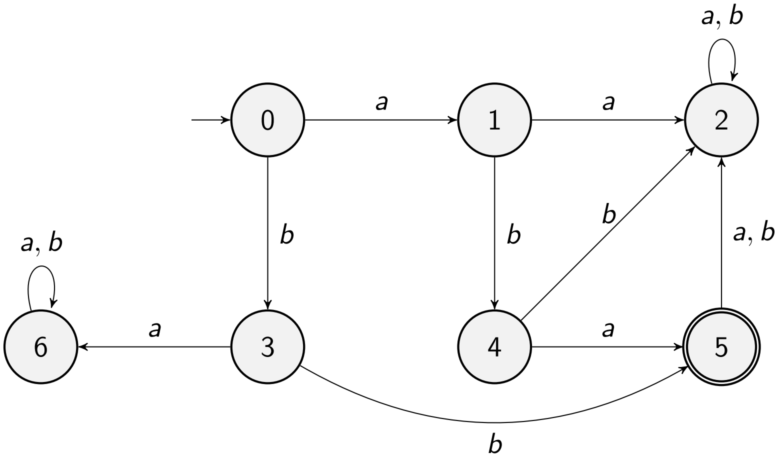

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Let L be a finite set. Let the longest string in L be of length n.

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This will take $\sim 2^n$ states. For many finite sets can do it with far fewer states.

DFA Intuitively

1. A DFA reads the input a letter at a time and never looks at it again. So one-scan.
2. A DFA only has a finite number of states, so $O(1)$ memory.
3. Contrast:
   3.1 A DFA can keep track of $\#_a(w) \pmod{17}$.
   3.2 A DFA cannot keep track of $\#_a(w)$.

DFA Formally

Def A **DFA** is a tuple $(Q, \Sigma, \delta, s, F)$ where:

1. Q is a finite set of **states**.
2. Σ is a finite **alphabet**.
3. $\delta : Q \times \Sigma \to Q$ is the **transition function**.
4. $s \in Q$ is the **start state**.
5. $F \subseteq Q$ is the set of **final states**.

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Def Let $L \subseteq \Sigma^*$. If there exists a DFA M such that $L(M) = L$ then L is **regular**.

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- If it's a particular example and not too many states, like those drawn a few slides ago, then draw it.

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- If it's a particular example and not too many states, like those drawn a few slides ago, then draw it.
- If it is many states or a general case (next slide) then give the transition table (the definition of δ).

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Number of states is nm. Is there a DFA for this lang with a smaller DFA?

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No.

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Number of states is nm. Is there a DFA for this lang with a smaller DFA?
No. We may prove this later in the term.