Homework 5 Morally Due Mar 5

1. (40 points)

   (a) (0 points) READ the $R(i, j, k)$ method (on the course webpage under notes) for GIVEN a DFA, produce a REGEX for the same language. NOTE that it is a DYNAMIC PROGRAMMING algorithm. (That means it's a recursion, but done from the bottom up instead of top down.)

   (b) (20 points) Write the $R(i, j, k)$ algorithm as a RECURSIVE program.

   (c) (0 points) READ up on Memoization (there is a nice Wikipedia entry on it, plus it is in many algorithms texts and on the web in other places).

   (d) (20 points) Write the $R(i, j, k)$ algorithm as a MEMOIZATION program which has the benefits of both recursion and dynamic programming!

SOLUTION TO PROBLEM ONE

Omitted.

GO TO NEXT PAGE!!!!!!!!!!!

2. (40 points) For each of the following state if it's REGULAR or NOT REGULAR. If it's REGULAR then give a DFA or REGEX for it. If it's NOT REGULAR then prove that.

Recall that $n_a(w)$ is the Number of a's in w. Also, recall that N (natural numbers) denotes the set of *nonnegative* integers.

(a) (8 points) (Alphabet is $\{a\}$.)

$$\{a^n a^n \mid n \in \mathsf{N}\}$$

(b) (8 points) (Alphabet is $\{a, b\}$.) Here, x^R denotes the reverse of a string (so $(aab)^R = baa$).

$$\{xyx^R \mid x, y \in \{a, b\}^*\}$$

(c) (8 points) (Alphabet is $\{a\}$.)

$$\{a^{\lceil \log_2(n+1) \rceil} \mid n \in \mathsf{N}\}$$

(d) (8 points) (Alphabet is $\{a, b\}$.)

$$\{a^{n^2} b^n \mid n \in \mathsf{N}\}$$

(e) (8 points) (Alphabet is $\{a, b\}$.)

$$\{a^m b^n \mid m, n \in \mathsf{N} \text{ AND } m \geq n^2\}$$

SOLUTION TO PROBLEM TWO

a) REGULAR: this is just $(aa)^*$

b) REGULAR: This is $\{a, b\}^*$. Note that if $y \in \{a, b\}^*$ then $eye^R = y$.

c) REGULAR: This is a^*.

d) This is NOT regular. Assume, by way of contradiction, that the lang is regular. There exists n_0 such that:

for all $w \in L$, $|w| \geq n_0$, there exists x, y, z such that

- $w = xyz$
- $y \neq e$
- $|xy| \leq n_0$
- for all $i \geq 0$, $xy^iz \in L$

Now take $a^{n^2}b^n$ long enough so that $n_0 < n^2$. So when this string is written as xyz, the y part is within the a^{n^2}.

$a^{n^2}b^n = xyz$ where $x = a^{n_1}$, $y = a^{n_2}$, $z = a^{n_3}b^n$

where $n_1 + n_2 + n_3 = n^2$. Note $n_2 > 0$.

Pump just once to get

$w = a^{n_1}a^{2n_2}a^{n_3}b^n = a^{n^2+n_2}b^n \notin L$,

a contradiction.

e) This is NOT regular. Assume, by way of contradiction, that the lang is regular. There exists n_0 such that:

for all $w \in L$, $w \geq n_0$, there exists x, y, z such that

- $w = xyz$
- $y \neq e$
- $|xy| \leq n_0$
- for all $i \geq 0$, $xy^iz \in L$

Now take $a^{n^2}b^n$ long enough so that $n_0 < n$. So when this string is written as xyz, the y part is within the a^{n^2}. (NOTE- I can pick whatever string in L I want. The PL says that EVERY string in L that is long enough can be pumped and stay in L.)

$a^{n^2}b^n = xyz$ where $x = a^{n_1}$, $y = a^{n_2}$, $z = a^{n_3}b^n$

where $n_1 + n_2 + n_3 = n^2$. Note $n_2 > 0$.

KEY- if we pump 2 or 3 or more times we are STILL in the language. So we pump 0 times to get

$$xy^0z = xz = a^{n_1+n_3}b^n$$

And note that $n_1 + n_3 = n^2 - n_2 < n^2$. so NOT in L.

3. (20 points) (NOTE- this is similar to HW03, Problem 3 which was harder than I intended. Hence we will revise those grades as follows:

   Grade on HW03, Problem 3 =

   $$\max\{(\text{Grade HW3, Prob 3}), (\text{Grade HW 5, Prob 3– This problem})\}$$

   And NOW onto the problem:

   Let

   $$L = \{a^n \mid n \not\equiv 0 \pmod{3811}\}.$$

   (a) (10 points) Prove that ANY DFA for L has to have ≥ 3811 states.

   (b) (10 ponts) Prove or Disprove: There is an NFA for L with < 3811 states.

SOLUTION TO PROBLEM THREE

a) Assume that there is a DFA for L with < 3811 states. Input a^{3811} to this DFA. In its run there must be a repeated state. Hence there exist numbers i and j with $1 \leq i < j \leq 3811$ such that

a^i and a^j both end up in state q.

Then $a^i a^{3811-j} = a^{3811+i-j} \neq a^{3811}$ and $a^j a^{3811-j} = a^{3811}$ both end up in the same state q. But the first string should be rejected and the second one accepted. This is a contradiction.

b) YES there is an NFA for L with MUCH LESS than 3811 states. Note that $3811 = 37 \times 103$.

Let $n \not\equiv 3811$. Note that n CANNOT be BOTH $\equiv 0 \pmod{103}$ and $\equiv 0 \pmod{37}$ (if it was then it would be $\equiv 0 \pmod{3811}$). Hence either

- There exists $i \in \{1, \ldots, 36\}$, $n \equiv i \pmod{37}$, or
- There exists $i \in \{1, \ldots, 102\}$, $n \equiv i \pmod{103}$.

Hence your NFA does the following: two e-transistions from the start state: (1) one of them goes to a DFA that accepts iff $n \not\equiv 0 \pmod{37}$

(this takes 37 states), (2) the other goes to a DFA that accepts iff $n \not\equiv 0 \pmod{103}$ (this takes 103 states)

Hence there is an NFA for L with 140 states plus the start state, so 141 states MUCH less than 3811.

END OF SOLUTION TO PROBLEM THREE