Homework 5 Morally Due Mar 5
THIS HOMEWORK IS TWO PAGES LONG!!!!!!!!!!!!!!!

1. (40 points)

   (a) (0 points) READ the $R(i,j,k)$ method (on the course webpage under notes) for GIVEN a DFA, produce a REGEX for the same language. NOTE that it is a DYNAMIC PROGRAMMING algorithm. (That means it's a recursion, but done from the bottom up instead of top down.)

   (b) (20 points) Write the $R(i,j,k)$ algorithm as a RECURSIVE program.

   (c) (0 points) READ up on Memoization (there is a nice Wikipedia entry on it, plus it is in many algorithms texts and on the web in other places).

   (d) (20 points) Write the $R(i,j,k)$ algorithm as a MEMOIZATION program which has the benefits of both recursion and dynamic programming!

GO TO NEXT PAGE!!!!!!!!!!!

2. (40 points) For each of the following state if it's REGULAR or NOT REGULAR. If it's REGULAR then give a DFA or REGEX for it. If it's NOT REGULAR then prove that.

   Recall that $n_a(w)$ is the Number of a's in w. Also, recall that N (natural numbers) denotes the set of *nonnegative* integers.

   (a) (8 points) (Alphabet is $\{a\}$.)

   $$\{a^n a^n \mid n \in \mathsf{N}\}$$

   (b) (8 points) (Alphabet is $\{a, b\}$.) Here, x^R denotes the reverse of a string (so $(aab)^R = baa$).

   $$\{xyx^R \mid x, y \in \{a, b\}^*\}$$

   (c) (8 points) (Alphabet is $\{a\}$.)

   $$\{a^{\lceil \log_2(n+1) \rceil} \mid n \in \mathsf{N}\}$$

   (d) (8 points) (Alphabet is $\{a, b\}$.)

   $$\{a^{n^2} b^n \mid n \in \mathsf{N}\}$$

   (e) (8 points) (Alphabet is $\{a, b\}$.)

   $$\{a^m b^n \mid m, n \in \mathsf{N} \text{ AND } m \geq n^2\}$$

3. (20 points) (NOTE- this is similar to HW03, Problem 3 which was harder than I intended. Hence we will revise those grades as follows:

   Grade on HW03, Problem 3 =

   $$\max\{(\text{Grade HW3, Prob 3}), (\text{Grade HW 5, Prob 3– This problem})\}$$

   And NOW onto the problem:

   Let

   $$L = \{a^n \mid n \not\equiv 0 \pmod{3811}\}.$$

   (a) (10 points) Prove that ANY DFA for L has to have ≥ 3811 states.

   (b) (10 ponts) Prove or Disprove: There is an NFA for L with < 3811 states.