

## PROJECT

If you do this project AND end up with an F or D in the course then I will grade it and MAY use your grade to BUMP your grade up (from an F to a D-, from a D to a C-). Throughout this document “prove” means “give a construction and discuss why it works.” What you hand in must be TYPED or VERY GOOD HANDWRITING.

DUE the DAY of the final. Absolute Deadline.

HINT: START early. Feel free to get help from me or the TA.

1. (0 points but you have to answer) What is your name? Write it clearly.
2. Let  $L$  be regular. Prove or Disprove or state that it is unknown to science. (You may use the equivalence of DFA, NFA's, and Regular Expressions).
  - (a)  $\bar{L}$  is regular.
  - (b)  $L^*$  is regular.
3. Let  $L$  be in P. Prove or Disprove or state that it is unknown to science.
  - (a)  $\bar{L}$  is in P.
  - (b)  $L^*$  is in P.
4. Let  $L$  be in NP. Prove or Disprove or state that it is unknown to science.
  - (a)  $\bar{L}$  is in NP.
  - (b)  $L^*$  is in NP.
5. Let  $L$  be decidable. Prove or Disprove or state that it is unknown to science.
  - (a)  $\bar{L}$  is decidable.
  - (b)  $L^*$  is decidable.
6. Let  $L = \{a^i : i \neq 2017\}$ . Give an NFA for  $L$  with  $\leq 500$  states (you can probably do it in far less than 500).
7. Let

$$L = \{a^{2n}b^{3n} : n \in \mathbb{N}\}$$

- Show that  $L$  is not regular using the extended Pumping Lemma.
  - Show that  $L$  is not regular using communication complexity.
8. (a) Describe the reduction of SAT to CLIQUE. That is, describe how you would take a formula  $\phi$  (we can assume its in CNF form) and from it get a graph  $G$  and a number  $k$  such that

$$\phi \in SAT \text{ iff } (G, k) \in CLIQUE.$$

- (b) Use the answer to part 1 to find a graph  $G$  and a number  $k$  such that

$$(x_1 \vee x_2) \wedge (\neg x_1 \vee x_3) \wedge (x_1 \vee x_2 \vee x_3) \in SAT$$

iff

$$(G, k) \in CLIQUE.$$