452 MIDTERM

Do not open this exam until you are told. Read these instructions:

1. This is a closed book exam, though ONE sheet of notes is allowed. **No calculators, or other aids are allowed**. If you have a question during the exam, please raise your hand.
2. There are 4 problems which add up to 100 points. The exam is 1 hours 15 minutes.
3. For each question show all of your work and **write legibly**. **Clearly indicate** your answers. No credit for illegible answers.
4. After the last page there is paper for scratch work. If you need extra scratch paper **after** you have filled these areas up, please raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper **will not** be graded.
5. Please write out the following statement: "*I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*"

6. Fill in the following:

NAME :
SIGNATURE :
SID :
SECTION NUMBER :

SCORES ON PROBLEMS

Prob 1:	
Prob 2:	
Prob 3:	
Prob 4:	
TOTAL	

1. (25 points) The alphabet is $\{a, b, c\}$. Recall that $\#_a(w)$ is how many a's are in w (similar for $\#_b$ and $\#_c$). Let L be the set of strings w such that

   EITHER

   $\#_a(w) \equiv 1 \pmod 2$

   OR

   $\#_b(w) \equiv 2 \pmod 3$

   OR

   $\#_c(w) \equiv 3 \pmod 4$.

   (a) (10 points) Describe a DFA for L. How many states does it have?

   (b) (15 points) Describe an NFA for L. How many states does it have (it should be LESS THAN the number of sates in the DFA)?

2. (25 points). A *JOSH DFA* is $(Q, \Sigma, \delta, S, F)$ where

   - Q is a set of states (as usual)
   - Σ is the alphabet (as usual)
   - $\delta : Q \times \Sigma \rightarrow Q$ (as usual)
   - $S \subseteq Q$ (THIS is what is different- S is a SET of states, not just one state)
   - $F \subseteq Q$.

   Let M be a JOSH DFA. A string x is ACCEPTED by M if THERE EXISTS a state $s \in S$ such that if you run M on x STARTING in s you end up in a state in F.

   A language L is JOSH-regular if there is a JOSH DFA M such that $x \in L$ iff $M(x)$ accepts.

   Show that if L is JOSH-regular THEN L is regular.

3. (25 points) We use the WS1S convention which is, of course, the CORRECT convention to use no matter what anyone says. Recall that, for example $(2, 5, \{0, 3, 5, 6\})$ is represented as follows:

$$\begin{array}{ccccccc} 0 & 0 & 1 & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 1 & * \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}$$

where the $*$ can be either 0 or 1.

NOW for the problem: Let

$$L_1 = \{(x, y, X) : x \in X \wedge y \notin X\}$$

$$L_2 = \{(x, y, X) : x \notin X \vee y \in X\}$$

(a) Draw a DFA for L_1. Label each state ACCEPT, REJECT, or BAD FORMAT.

(b) Draw a DFA for L_2. Label each state ACCEPT, REJECT, or BAD FORMAT.

4. (25 points) Let

$$L = \{w : \#_a(w) = 2\#_b(w)\}$$

(so if there are 40 b's then there are 80 a's).

(a) (10 points) Show that L is NOT regular using the Extended Pumping Lemma.

(b) (15 points) Show that L is NOT regular using Communication Complexity.

Scratch Paper