CMSC 250 Second Midterm

1. This is an open-everything exam. You can use anything except ask another person. **Caution:** if you copy from the web or elsewhere mindlessly you will probably get it wrong.

2. There are 3 problems which add up to 70 points. Recall that you already did 30 points of this midterm take home.

3. The exam is April 8 from 8:00PM until 10:15PM unless you have contacted me to make other arrangements. So the exam is 2 hours and 15 minutes

4. For each question show all of your work and **use LaTeX or write VERY NEATLY**. **Clearly indicate** your answers. No credit for illegible answers.

5. Please write out the following statement: *I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*

1. (25 points) Prove that $7^{1/3}$ is NOT rational. You must state and prove carefully any lemmas you use.

   You can do this problem on this page and the next page.

2. (25 points) Let a_n be defined as follows.

   $a_1 = 11$

   $(\forall n \geq 2)[a_n = a_{\lfloor n^{3/4} \rfloor} + a_{\lfloor n^{1/4} \rfloor} + 15]$

   Show by strong induction that

   $(\forall n \geq 1)[a_n \equiv 11 \pmod{13}]$

   Include Base Case, IH, and IS.

   You can do this problem on this page and the next page.

3. (20 points) Note that $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$.

Also note that $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{42} = 1$.

We obtained the 2nd equation from the 1st by noticing that $\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$.

This equation is a special case of the equation $\frac{1}{d} = \frac{1}{d+1} + \frac{1}{d(d+1)}$

which can be proven algebraically (it does not need a proof by induction).

OKAY, now finally the problem.

Show that, for all $n \geq 3$, there exists natural numbers $d_1 < \cdots < d_n$ such that

$\frac{1}{d_1} + \cdots + \frac{1}{d_n} = 1$.

(**Hint** Do this by induction on n and use

$$\frac{1}{d} = \frac{1}{d+1} + \frac{1}{d(d+1)}.$$

)