Homework 1

Morally due Mon Feb 8, 9:00AM. DEAD CAT Wed Feb 10 9:00AM

THE HW IS FIVE PAGES LONG!!!!!!!!!!!!!

1. (20 points) You can use Wolfram Alpha on this problem for the calculations. In the year 2030 there are 30 students taking CMSC 250H. There are three TAs: Alice, Bob, and Carol. The students are broken into three project groups, each lead by one of the TA's.

   (a) (6 points) Assume Alice will take 10 students, Bob will take 10 students, Carol will take 10 students.

   How many ways can the students be assigned to projects? Explain your answer. Give the number in terms of BOTH factorials and as an actual number.

   (b) (7 points) Assume Alice will take 5 students, Bob will take 10 students, Carol will take 15 students.

   How many ways can the students be assigned to projects? Explain your answer. Give the number in terms of BOTH factorials and as an actual number.

   (c) (7 points) In the year 2040 there are n students taking CMSC 250H. There are k TAs: $A_1, \ldots, A_k$. The students are broken into k project groups, each lead by one of the TA's.

   Assume A_1 will take n_1 students, ..., A_k will take n_k students (Note that $n_1 + \cdots + n_k = n$.)

   How many ways can the students be assigned to projects?

   (d) (0 points but DO IT anyway since we are all here to learn) Speculate: What should $n_1, \ldots, n_k$ be to maximize the number of ways students can be assigned to projects?

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2. (20 points) Use a combinatorial argument (NOT algebraic) to show that if $S = a + b + c$ then

$$\frac{S!}{a!b!c!} = \frac{(S-1)!}{(a-1)!b!c!} + \frac{(S-1)!}{a!(b-1)!c!} + \frac{(S-1)!}{a!b!(c-1)!}$$

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3. (20 points)

   (a) (10 points) Fill in the blanks in the following statement. Describe your reasoning. BLANK will be a function of k, n.

   If $A \subseteq \{1, \ldots, n\}$ and $|A| = k$ then at least BLANK subsets of A have the same SUM.

   (b) (10 points) Write a program that, on input k, n determines (1) the most number of subsets of size k that have the same sum which we denote m, and (2) that sum which we denote s.

   EXAMPLE: If $n = 4$ and $k = 2$ then

   $1 + 2 = 3$

   $1 + 3 = 4$

   $1 + 4 = 5$

   $2 + 3 = 5$

   $2 + 4 = 6$

   $3 + 4 = 7$.

   Note that 5 occurs twice and everything else occurs once, so $m = 2$ and $s = 5$.

   For $n = 10$ and $k = 2, \ldots, 9$ run your program and present a table of the following information.

k	q	m	$m - q$	s
2				
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
9				

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4. (20 points-5 points each) Bill makes his glirand a lunch every day. It has

   (1) Sandwich: egg salad OR peanut butter OR tuna fish OR turkey OR ham.

   (2) Fruit: apple OR orange OR grapefruit OR coconut.

   (3) Snack: applesauce OR pretzels OR cheese.

   (4) Drink: apple juice OR beer.

   And NOW finally the questions: Each question below is separate.

   (a) How many ways can Bill make his glirand's lunch?

   (b) glirand says *I do not want to have BOTH and Apple and Applesauce* NOW how many ways can Bill make his glirand's lunch?

   (c) glirand says *I WANT to have two of the three Apple-things, but NOT all three.* NOW how many ways can Bill make his glirand's lunch?

   (d) glirand says *IF you give me a turkey or ham sandwich then you MUST give me a cheese snack.* NOW how many ways can Bill make his glirand's lunch?

   (e) (0 points) glirand is a permutation of what Bill really calls his wife. What does Bill call his wife?

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5. (20 points)

   (a) Do this problem from first principles, not using the formula. What is the coefficient of $v^2w^2x^2y^2z^2$ in the expansion of

   $$(v + w + x + y + z)^{10}.$$

   (As a sanity check see that it agrees with the formula.)