# CMSC 132: Object-Oriented Programming II

## UNDIRECTED GRAPHS

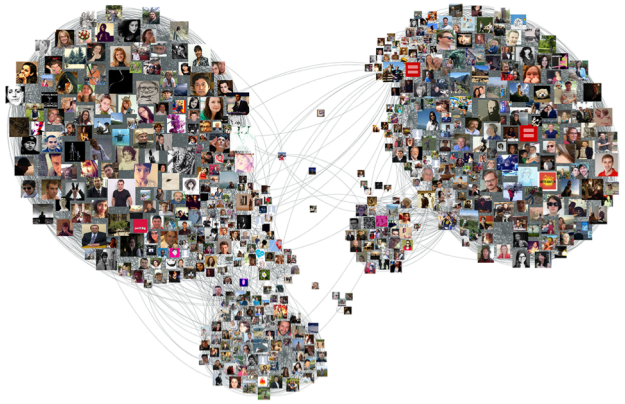
Graphs slides are modified from COS 126 slides of Dr. Robert Sedgewick.

# Undirected Graphs

Graph: Set of vertices connected pairwise by edges.

# Undirected Graphs

- Why study graph algorithms?
  - Thousands of practical applications.
  - Hundreds of graph algorithms known.
  - Interesting and broadly useful abstraction.
  - Challenging branch of computer science and discrete math.

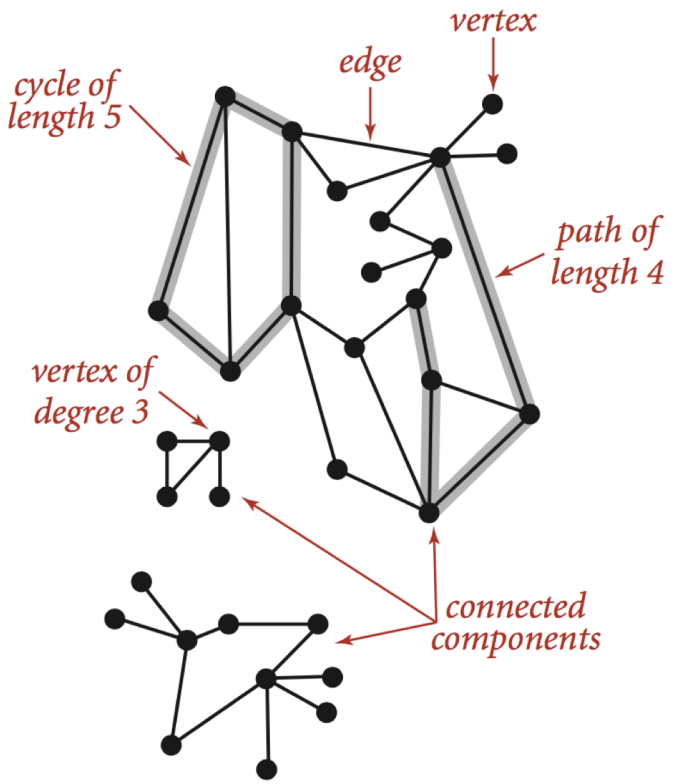



# Graph Applications

| Graph | Vertex | Edge |
|---|---|---|
| communication | Telephone, computer | fiber optic cable |
| circuit | gate, register, processor | wire |
| mechanical | joint | rod, beam, spring |
| financial | stock, currency | transactions |
| transportation | street intersection, airport | highway, airway route |
| internet | class C network | connection |
| game | board position | legal move |
| social relationship | person, actor | friendship, movie cast |
| chemical compound | molecule | bond |

# Graph Terminology

- Path:
  - Sequence of vertices connected by edges.
- Cycle
  - Path whose first and last vertices are the same.
- Two vertices are connected if there is a path between them.



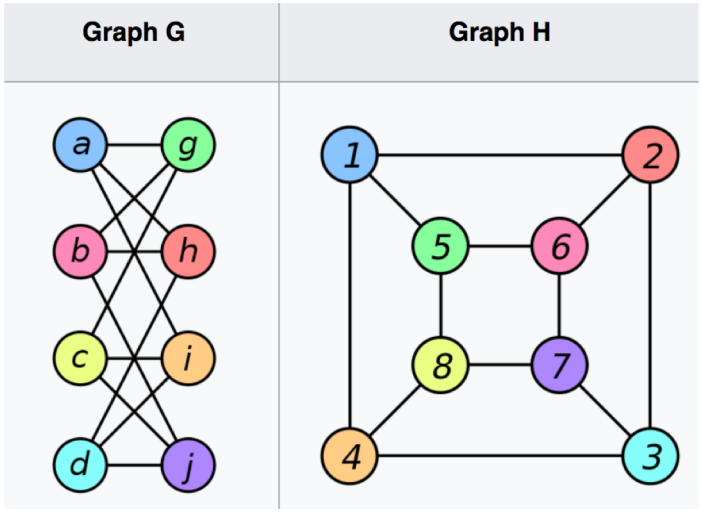
# Some graph-processing problems

- Path:
  - Is there a path between s and t ?
- Shortest path.
  - What is the shortest path between s and t ?
- Cycle.
  - Is there a cycle in the graph?
- Euler tour.
  - Is there a cycle that uses each edge exactly once?
- Hamilton tour.
  - Is there a cycle that uses each vertex exactly once.
- Connectivity.
  - Is there a way to connect all of the vertices?

# Some graph-processing problems

- MST.
  - What is the best way to connect all of the vertices?
- Biconnectivity.
  - Is there a vertex whose removal disconnects the graph?
- Planarity.
  - Can you draw the graph in the plane with no crossing edges
- Graph isomorphism.
  - Do two adjacency lists represent the same graph?

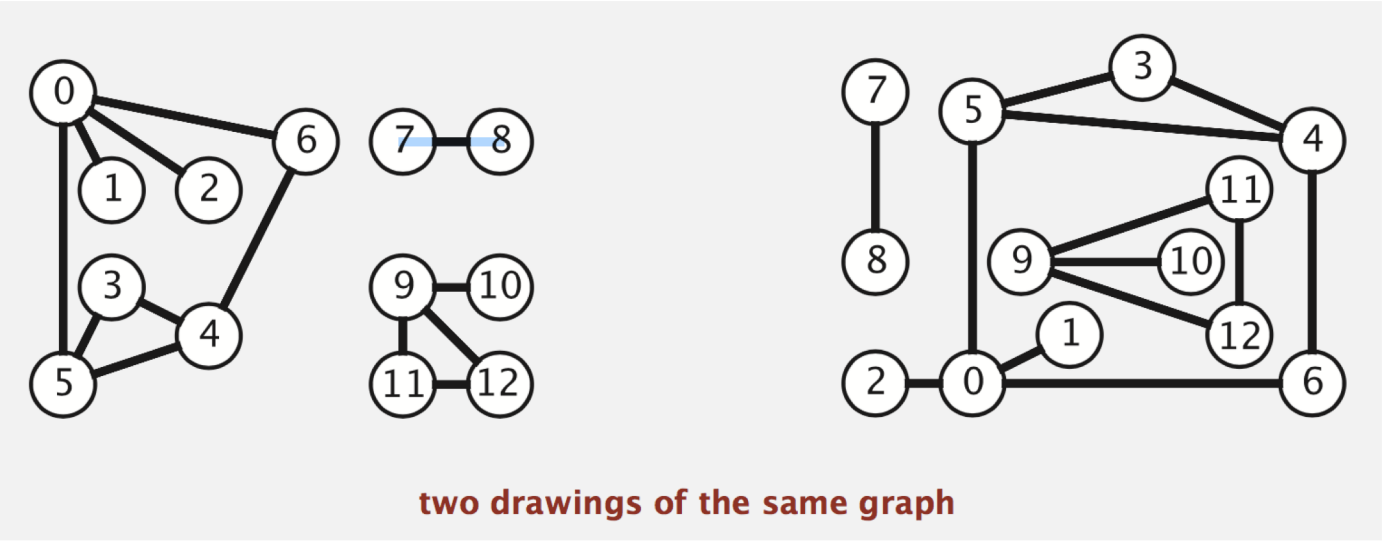
Challenge. Which of these problems are easy? difficult? intractable?



# Graph representation

## Graph drawing

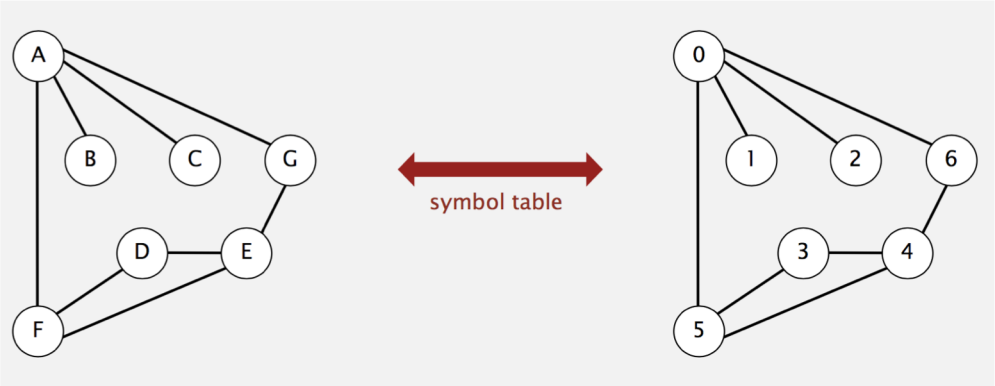
- Provides intuition about the structure of the graph.

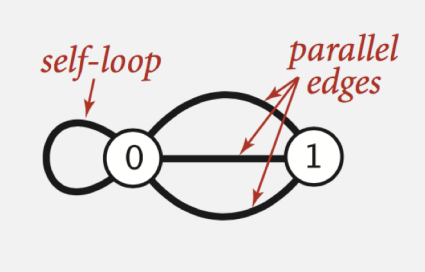



two drawings of the same graph

# Graph representation

- Vertex representation:
  - use integers between 0 and $V - 1$.
- Applications: convert between names and integers with symbol table.



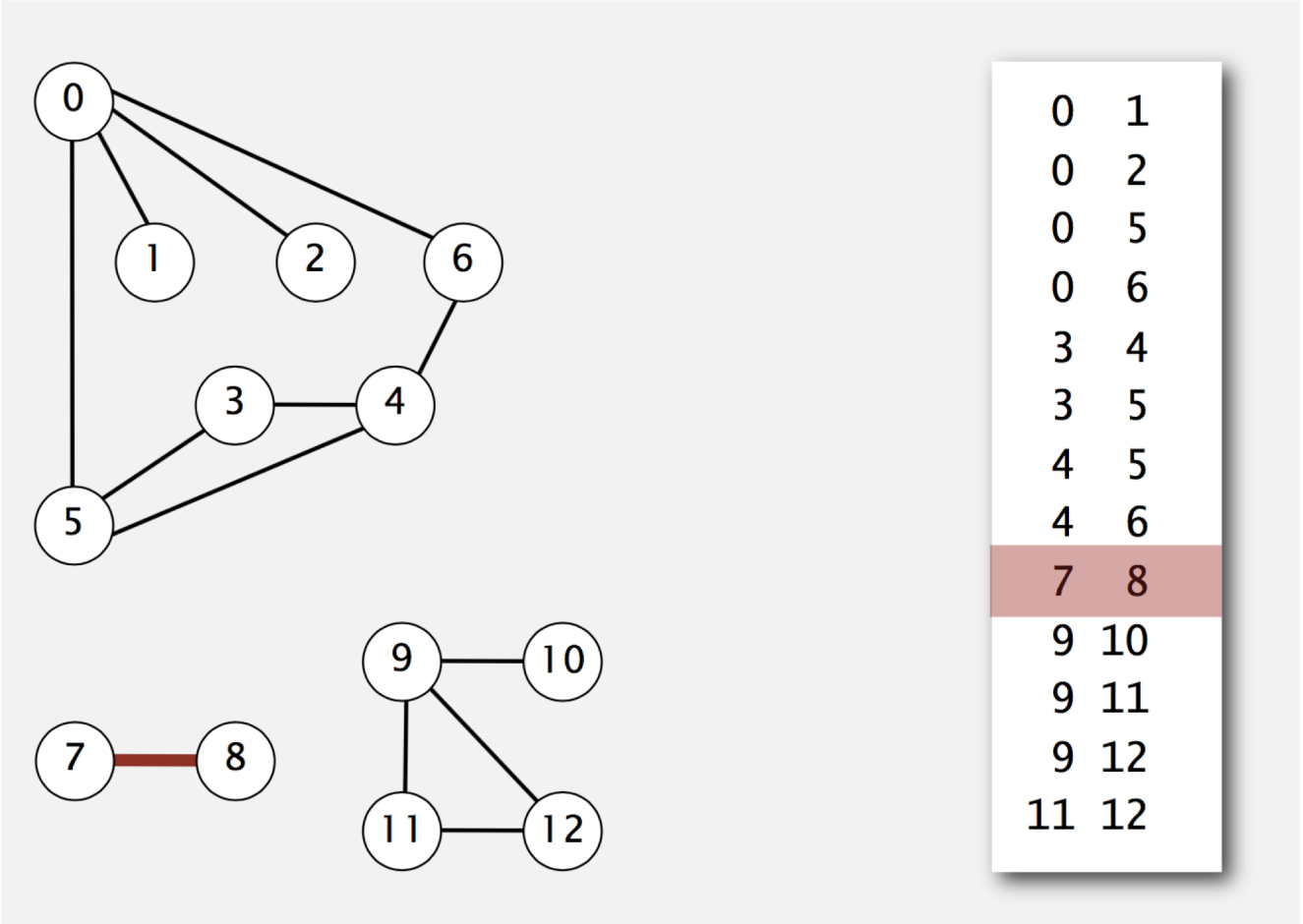

No self loop,
No parallel edges

# Graph Class

```
public class Graph{
   Graph(int V)  //create an empty graph with V
   void addEdge(int v, int w)  //add an edge v-w
   Iterable<Integer>adj(int v) //vertices adjacent to v
   int V()    //number of vertices
   int  E()   //number of edges
   String toString()  //string representation
}
```

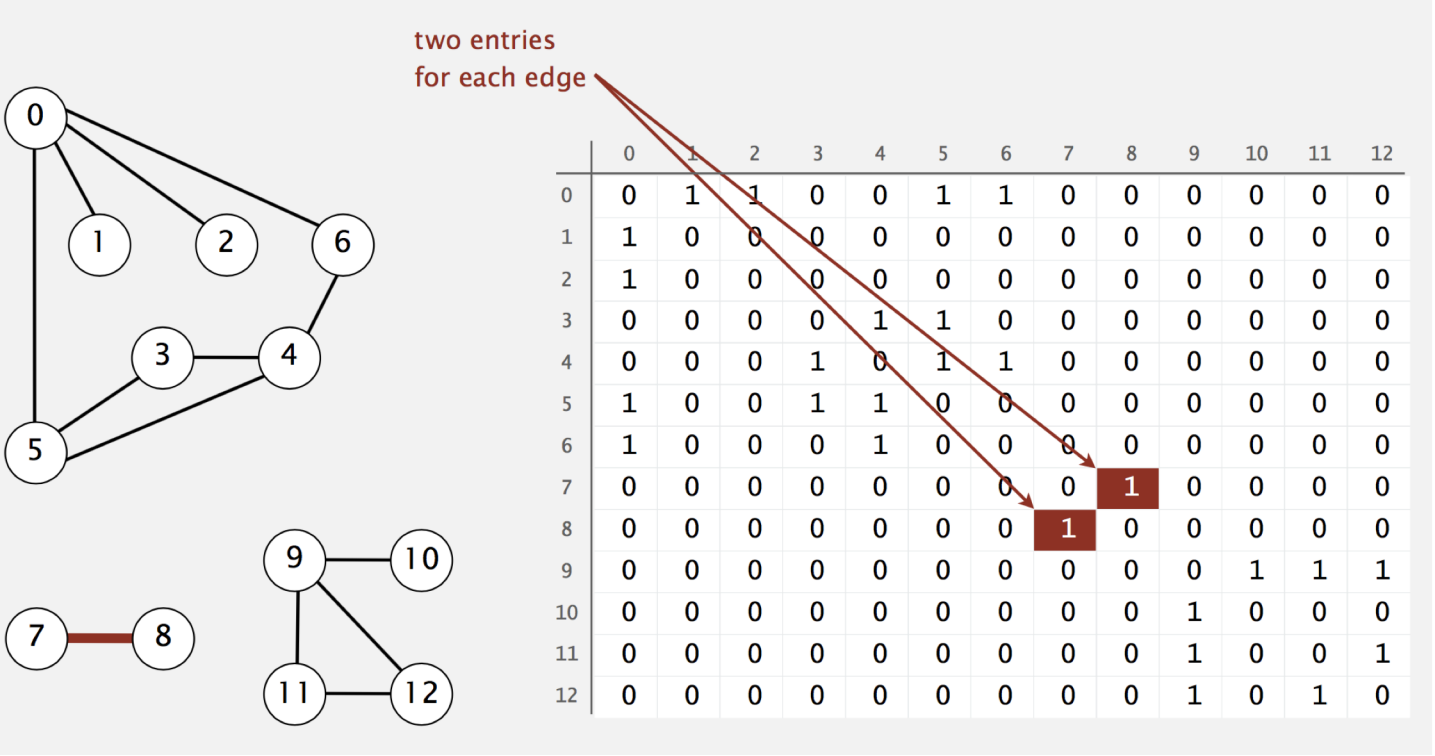
# Set-of-edges graph representation

- Maintain a list of the edges (linked list or array).



# Adjacency-matrix graph representation

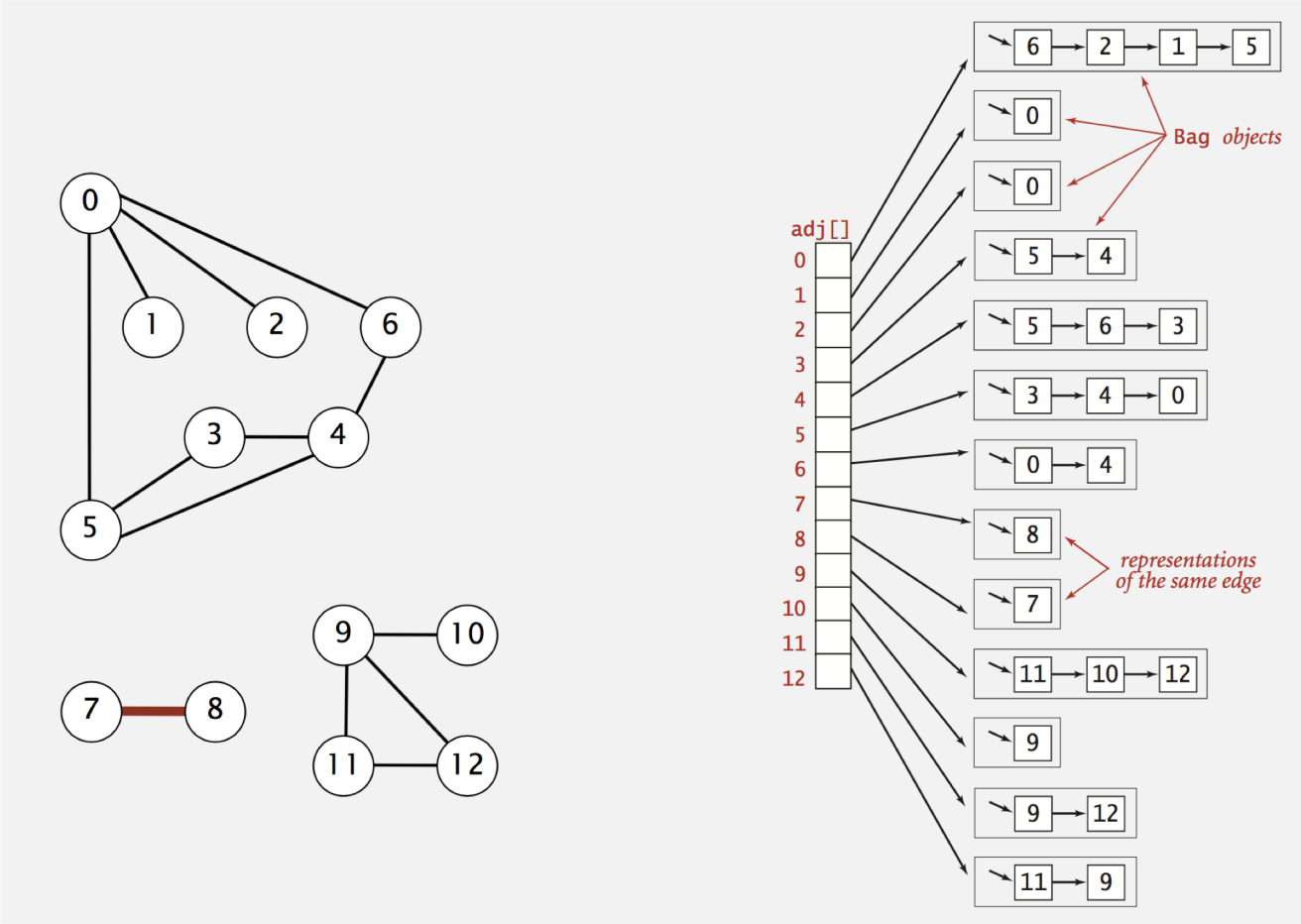
- Maintain a two-dimensional *V*-by-*V* boolean array;
- for each edge *v*–*w* in graph:
  - adj[v][w] = adj[w][v] = true.




| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |

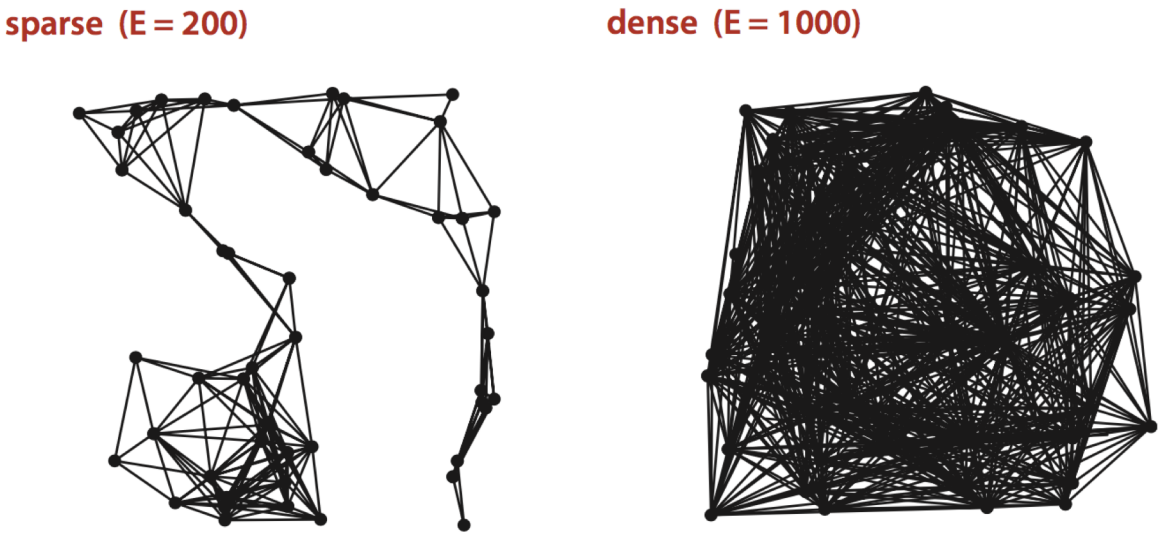
# Adjacency-list graph representation

- Maintain vertex-indexed array of lists.



# Graph representation

- In practice. Use adjacency-lists representation.
  - Algorithms based on iterating over vertices adjacent to *v*.
- Real-world graphs tend to be sparse.




Two graphs (V = 50)

huge number of vertices,
small average vertex degree

# Graph representation

Comparisons of three different representations:

| representation | space | add edge | edge between v and w? | iterate over vertices adjacent to v? |
|---|---|---|---|---|
| list of edges | E | 1 | E | E |
| adjacency matrix | $V^2$ | 1 * | 1 | V |
| adjacency lists | E + V | 1 | degree(v) | degree(v) |

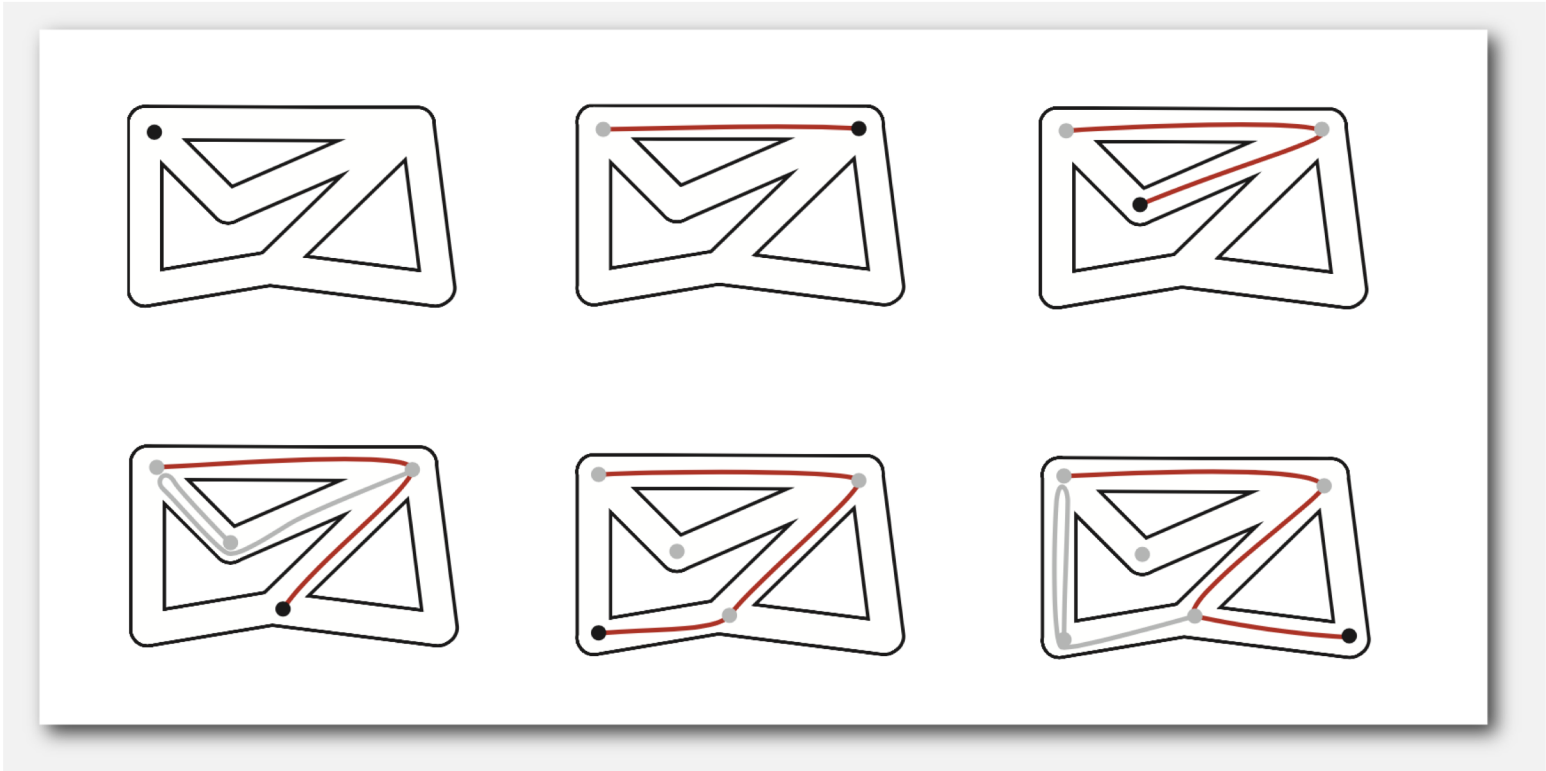
* disallows parallel edges

# Adjacency-list graph representation: Java implementation

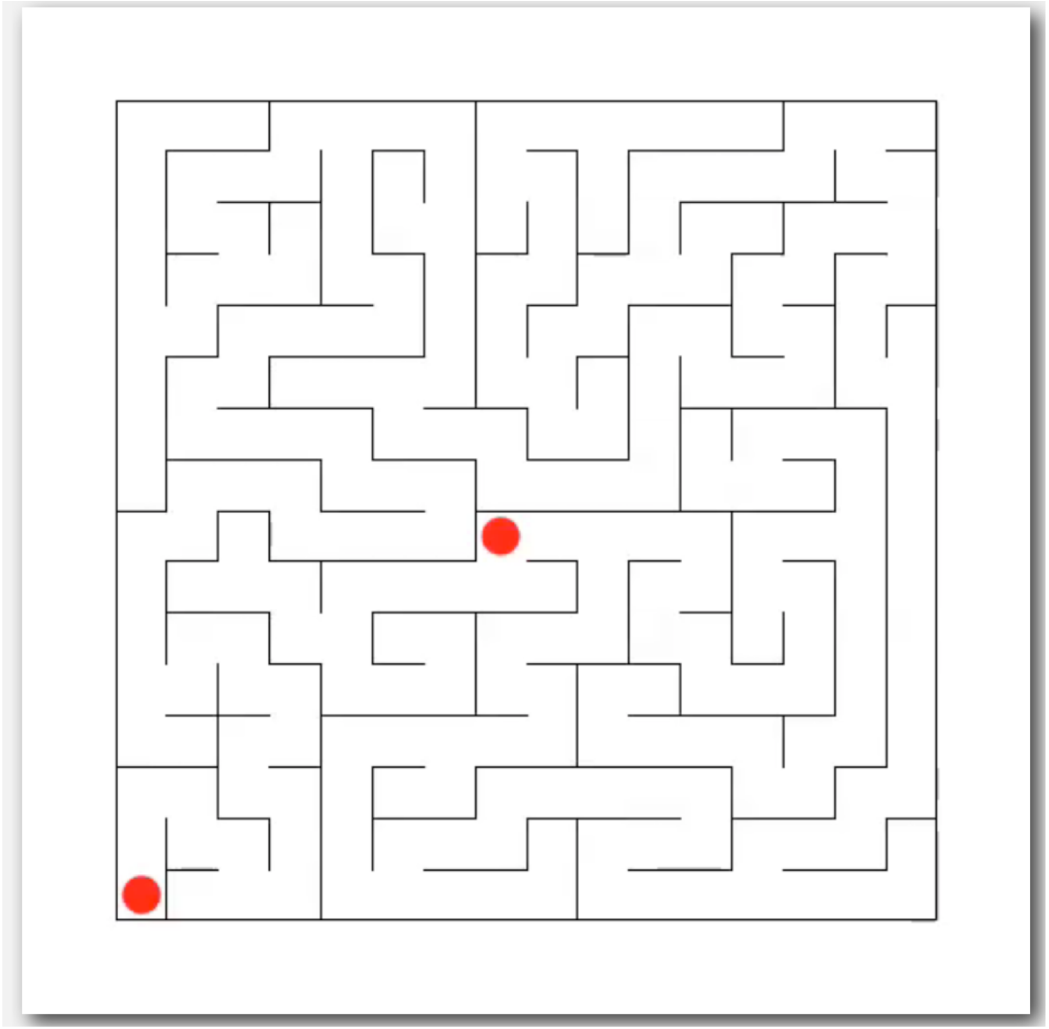
```
public class Graph{
  private final int V;
  private Bag<Integer>[] adj;
  public Graph(int V)  {
      this.V = V;
      adj = (Bag<Integer>[]) new Bag[V];
      for (int v = 0; v < V; v++)
         adj[v] = new Bag<Integer>();
      }
   public void addEdge(int v, int w) {
     adj[v].add(w);
     adj[w].add(v);
   }
    public Iterable<Integer> adj(int v) {
       return adj[v];
    }
}
```

# Graph Algorithms: Depth First Search

- Trémaux maze exploration Algorithm
  - Unroll a ball of string behind you.
  - Mark each visited intersection and each visited passage.
  - Retrace steps when no unvisited options



# Maze Exploration



# Depth First Search

Goal. Systematically search through a graph. Idea. Mimic maze exploration.

```
DFS (to visit a vertex v)
   Mark v as visited.
   Recursively visit all unmarked
        vertices w adjacent to v.
```

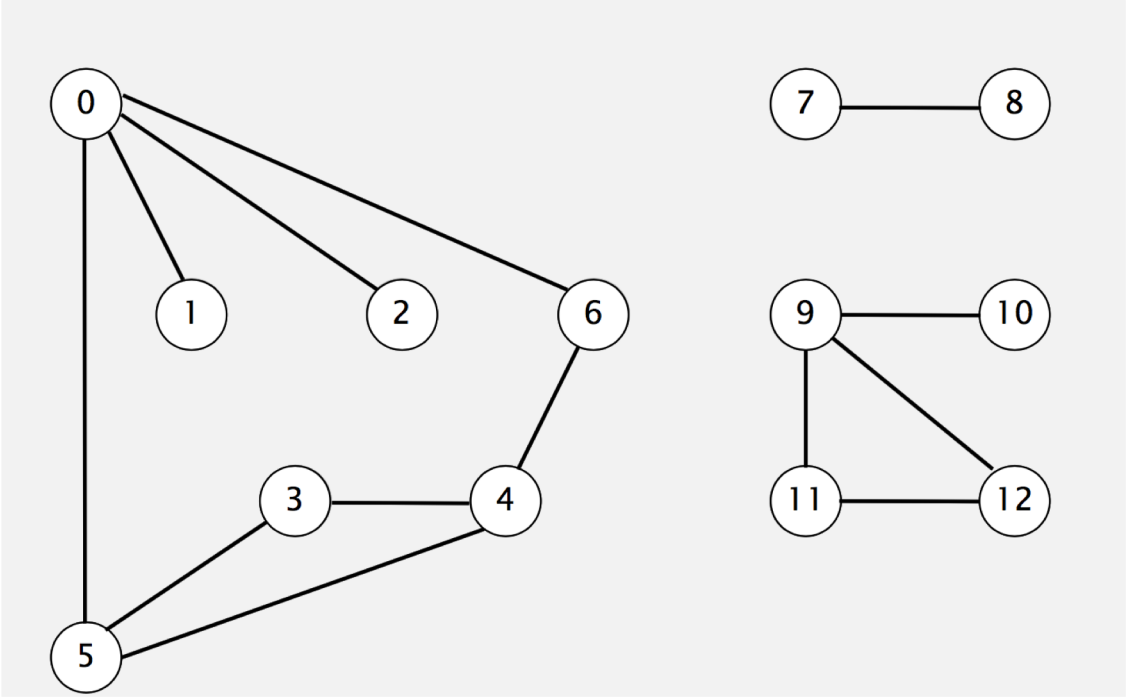
- Typical applications:
  - Find all vertices connected to a given source vertex.
  - Find a path between two vertices.

# DFS Demo

To visit a vertex *v* :

Mark vertex *v* as visited.

Recursively visit all unmarked vertices adjacent to *v*.



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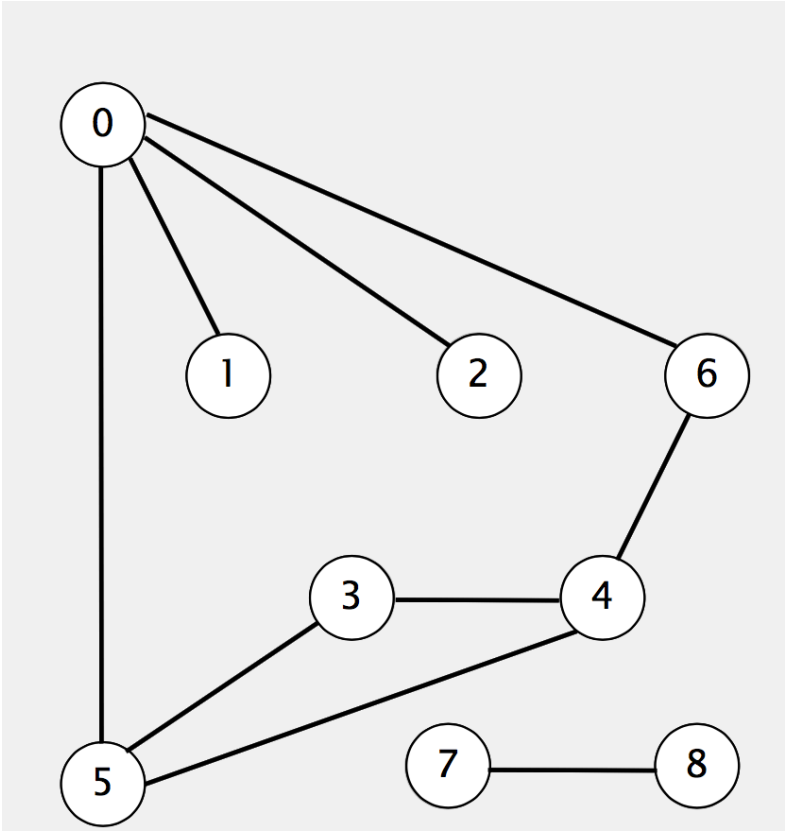


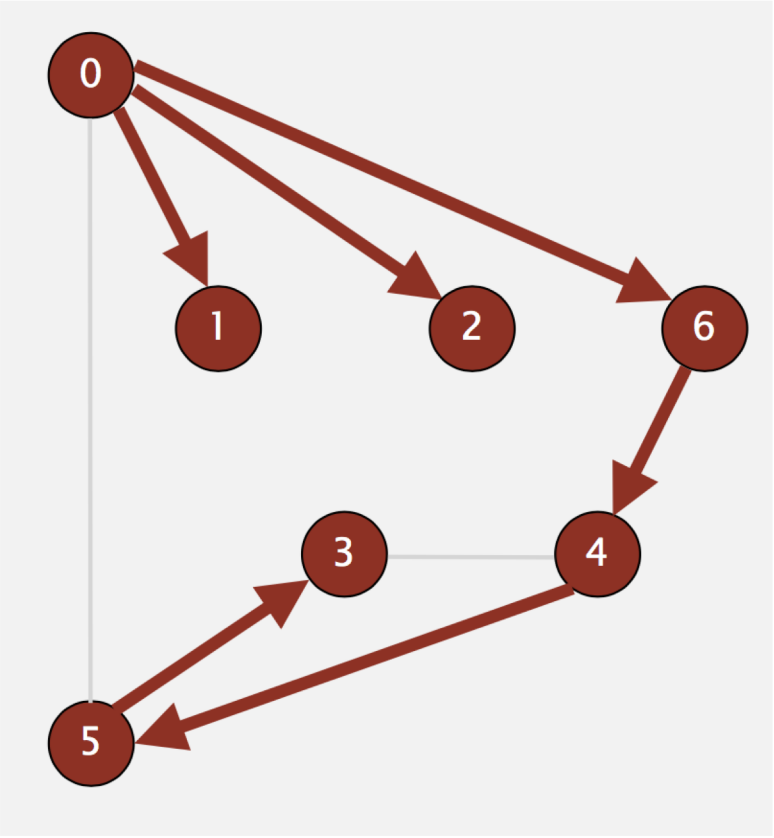

| V | **marked[]** | **edgeTo[v]** |
|---|---|---|
| 0 | | - |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |

# DFS Demo

To visit a vertex *v* :

Mark vertex *v* as visited.

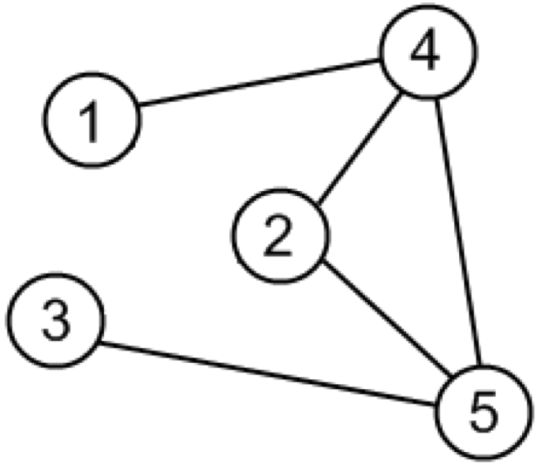
Recursively visit all unmarked vertices adjacent to *v*.




| V | **marked[]** | **edgeTo[v]** |
|---|---|---|
| 0 | T | - |
| 1 | T | 0 |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | |
| 8 | F | |

# Depth-first search

```
public class DepthFirstPaths {
  private boolean[] marked;
  private int[] edgeTo;
  private int s;
  public DepthFirstSearch(Graph G, int s) {
     ...
  dfs(G, s);
  }
  private void dfs(Graph G, int v) {
     marked[v] = true;
     for (int w : G.adj(v))
       if(!marked[w]) {
           dfs(G, w);
           edgeTo[w] = v;
       }
  }
}
```

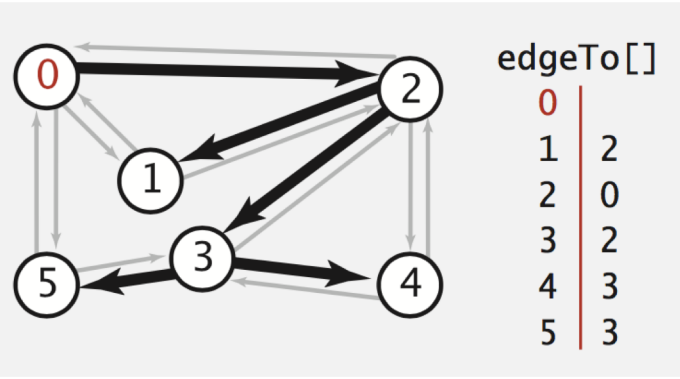


# Depth-first search properties

Proposition. After DFS, can find vertices connected to *s* in constant time and can find a path to *s* (if one exists) in time proportional to its length.

Pf. edgeTo[] is parent-link representation of a tree rooted at s.

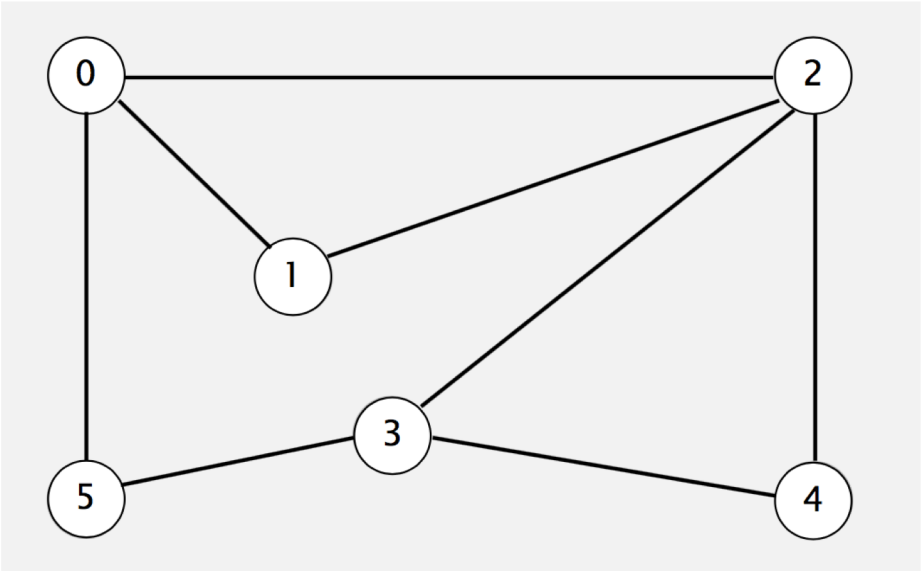
```
public boolean hasPathTo(int v) {return marked[v];}

public Iterable<Integer> pathTo(int v) {
   if (!hasPathTo(v)) return null;
   Stack<Integer> path = new Stack<Integer>();
   for (int x = v; x != s; x = edgeTo[x])
        path.push(x);
   path.push(s);
   return path;
}
```



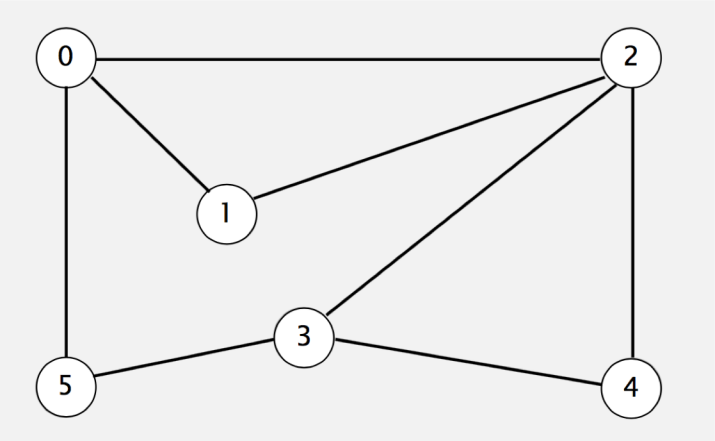
# Breadth-first search (BFS)

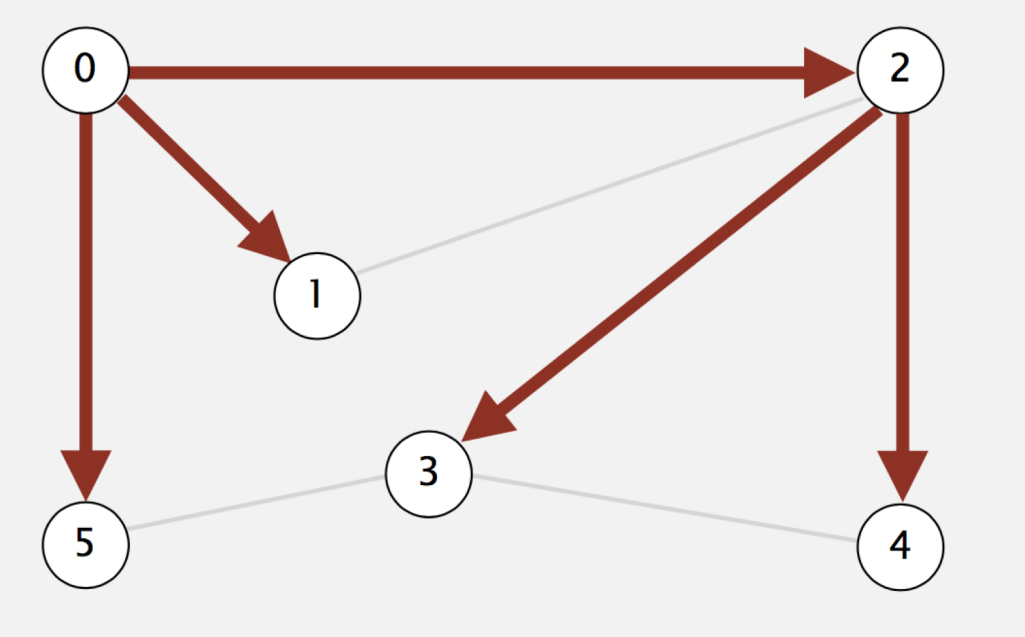
- BFS starts at a vertex and and explores the neighbor vertices first, before moving to the next level neighbors.

```
Repeat until queue is empty:
    Remove vertex v from queue.
    Add to queue all unmarked vertices
            adjacent to v and mark them.
```



# Breadth-first search (BFS)



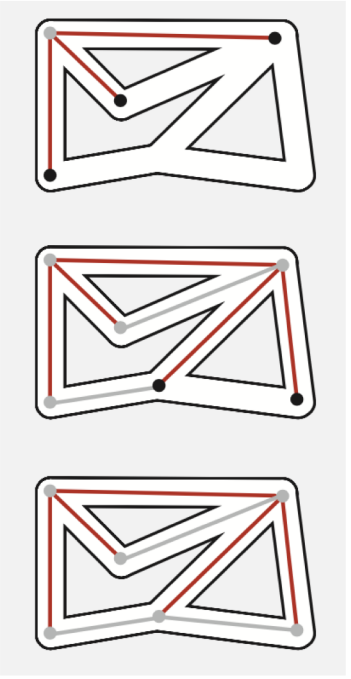

| v | edgeTo[] | distTo[] |
|---|---|---|
| 0 | – | 0 |
| 1 | 0 | 1 |
| 2 | 0 | 1 |
| 3 | 2 | 2 |
| 4 | 2 | 2 |
| 5 | 0 | 1 |

# Breadth-first search

Depth-first search: Put unvisited vertices on a stack.
Breadth-first search: Put unvisited vertices on a queue.
Shortest path: Find path from s to t that uses fewest number of edges.

**BFS (from source vertex s)**

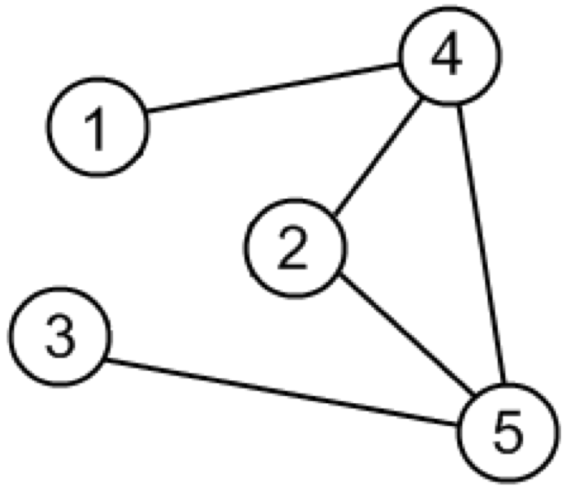
Put s onto a FIFO queue, and mark s as visited.
Repeat until the queue is empty:
remove the least recently added vertex v
add each of v's unvisited neighbors to the queue,
and mark them as visited.



Intuition: BFS examines vertices in increasing distance from s.

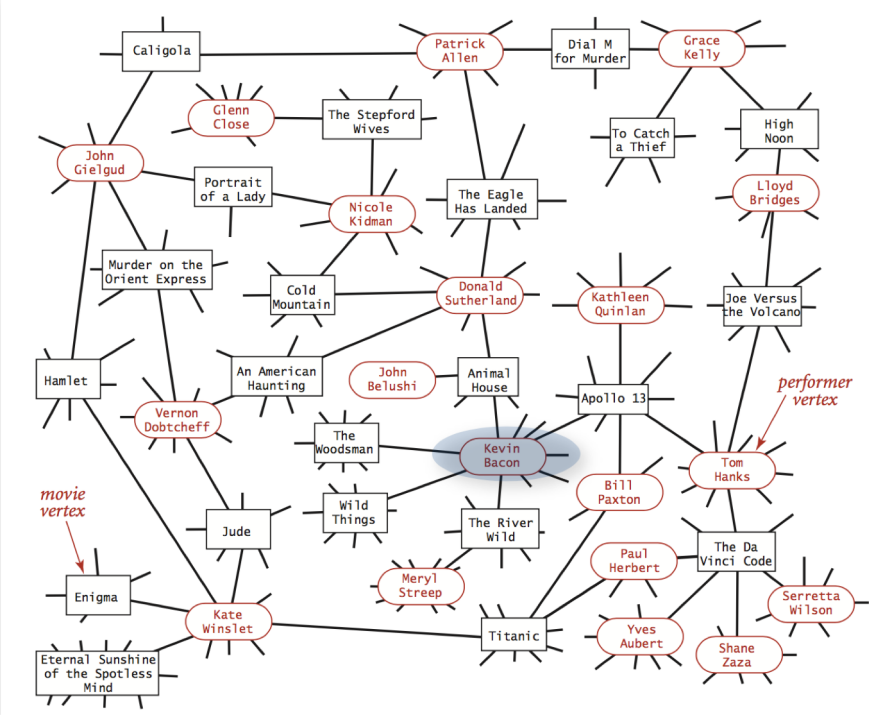
# Breadth-first search

```
public class BreadthFirstPaths {
   private boolean[] marked;
   private int[] edgeTo;
   ...
   private void bfs(Graph G, int s) {
     Queue<Integer> q = new Queue<Integer>();
     q.enqueue(s);
     marked[s] = true;
     while (!q.isEmpty()) {
        int v = q.dequeue();
        for (int w : G.adj(v)) {
           if (!marked[w]){
               q.enqueue(w);
               marked[w] = true;
               edgeTo[w] = v;
           }
        }
     }
  }
}
```



# BFS Application: Kevin Bacon Number

- Kevin Bacon graph
  - Include one vertex for each performer and one for each movie.
  - Connect a movie to all performers that appear in that movie
  - Compute shortest path from *s* = Kevin Bacon.



# Connected components

- Goal:
  - Partition vertices into connected components.

Connected components

Initialize all vertices v as unmarked.

For each unmarked vertex v, run DFS to identify all vertices discovered as part of the same component.