CMSC 451 - Algorithm Design
Lecture 1 - Introduction

About this course -

- Second course in algorithm design (after 351)
- Fundamental elements of algorithm design
  - design techniques (greedy, divide + conquer, ...)
  - proving correctness
  - analyzing running times
- Theoretical focus - no programming projects

Overview:

- Graph basics, DFS, + shortest paths
- Greedy algorithms
- Dynamic programming
- Network flows
- NP-Hardness + Approximation algorithms

Prerequisites - (CMSC 351)

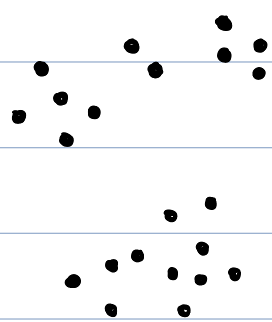
- Basic programming
- Discrete math - induction, sets, probability, ...
- Sorting + basic data structures
- Math - logs + exponentials, calculus, linear algebra

Algorithm Design:

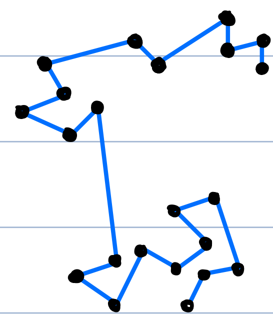
Given a well defined computational problem design an efficient procedure for solving it.

Example 1: Given a set of points in the plane compute a path of min length that visits all the points.

Input:

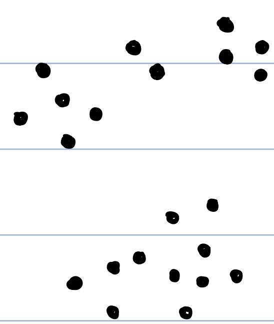


Output:

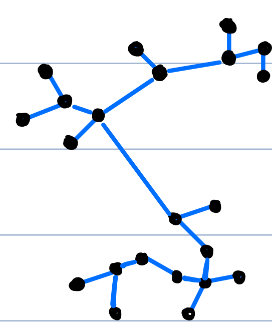


Example 2: Given a set of points in the plane compute a network of min length that connects all the points.

Input:



Output:



These are superficially quite similar, but computational complexities are quite different.

Example 1: Euclidean Traveling Salesman Problem
- NP-Hard
- Can be efficiently approximated

Example 2: Euclidean Minimum Spanning Tree
- Solvable exactly in $O(n^2)$ time
- Practical approximation in $O(n \log n)$ time

Measures of computational complexity -
- Running time
- Space

... as a function of input size (denoted n)

But many inputs of same size...
- Worst-case - max over all inputs of size n
- Average-case - expected case for inputs of size n from some distribution

→ More realistic, but a lot harder!

Asymptotic Notation:

- Simplify complex functions
- Focus on trend for large n
- Ignore constant factors

Example:

$$T(n) = 3.9n + 4.17 \cdot n\log n + 3.5n^2$$

$$\cong 3.9n + 4.17 \cdot n\log n + \boxed{3.5n^2}$$ — large n

$$\cong \cancel{3.9}n + \cancel{4.17} \cdot n\log n + 3.5\boxed{n^2}$$ — ignore const factors

$$\cong \cancel{3.5}n^2$$

$T(n)$ is $O(n^2)$ "on the Order of"

↳ Usually written $T(n) = O(n^2)$ but not formally correct. (But we'll do it anyway)

"O" is shorthand for asymptotically "$\leq$"

Also:

Θ	⟷	"$=$"	Big theta
o	⟷	"$<$"	Little o
Ω	⟷	"$\geq$"	Big Omega
ω	⟷	"$>$"	Little omega

Important Complexity Classes - For some constant $c > 0$

$\leq (\log n)^c = \log^c n$ — Polylogarithmic time

$\leq n^c$ — Polynomial time

$\leq c^n \quad (c > 1)$ — Exponential time

Asymptotic Ordering - for any $a, b, c > 0 \quad (c > 1)$

$$\log^a n \lesssim n^b \lesssim c^n$$

Summary:

- Basics of algorithm design
- Asymptotic Notation