NP-Complete Problems

Satisfiability (SAT)

Given a Boolean expression over n variables in conjunctive normal form, is there a way to assign values to the n variables that will make the entire expression true?

This general problem is NP-Complete.

NOTE: Even 3-SAT is NPC.

What does that mean?

NP-Complete

I will leave the finer details of the definition of NP-Completeness to CMSC 451, but we can discuss the relationship between NPC problems.

You can perform what's called a polynomial "conversion" from any NPC problem to any other NPC problem.

This means that if any NPC problem is ever solved in polynomial time, then *EVERY* NP-Complete problem can be solved in polynomial time!

Hamiltonian Cycles

Given an undirected graph, is there a cycle that visits every vertex exactly once?

If I present you with a graph and a cycle through it (the certificate), it is "easy" (polynomial runtime) to tell whether it is a Hamiltonian Cycle.

How can I convince you that there is ***not*** a Hamiltonian Cycle?

This problem is NP-Complete.

Hamiltonian Path U→V

Consider the problem of being given a graph, and two distinct vertices in the graph, and then having to say whether there is a Hamiltonian Path (every vertex visited exactly once) between the two points.

Is this in NP?

Is this in NPC?

NPC Reductions

In general, if:

- A is a problem known to be in NPC
- B is a new problem that is in NP

then if we can "restate" A as B (and technically also "restate" B as A) doing each of the conversions using "polynomial work" that means that B is also in NPC.

Super-Poly vs Sub-Exp