

Problem Set #9

CMSC 657

Instructor: Daniel Gottesman

Due on Gradescope, Thursday, Nov. 7, 2024, at 5:00 PM

Problem #1. Practice with Stabilizer Codes (30 pts.)

For this problem, consider the code with stabilizer S generated by:

$$\begin{array}{cccccccc} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 \\ Z_1 & Z_2 & Z_3 & Z_4 & Z_5 & Z_6 & Z_7 & Z_8 \\ X_1 & I & X_3 & I & Z_5 & Y_6 & Z_7 & Y_8 \\ X_1 & I & Y_3 & Z_4 & X_5 & I & Y_7 & Z_8 \\ X_1 & Z_2 & I & Y_4 & I & Y_6 & X_7 & Z_8 \end{array}$$

- (5 pts.) How many encoded qubits does this code have?
- (8 pts.) Find the error syndrome of the following errors: X_3 , Y_5 , X_3Y_5 , $X_3Y_5Z_8$.
- (8 pts.) Based *only* on the answers from the previous part and the fact that none of the Paulis in part b is in S , what can you say about the distance of this code?
- (9 pts.) Consider the Pauli $P = Y_1Z_2Y_3Z_4X_6X_8$. Is P in $N(S)$? Is P in S ? Is $-P$ in S ?

Problem #2. Error Syndromes and Cosets (30 pts.)

Recall that $N(S) = \{P \mid [P, M] = 0 \forall M \in S\}$. If H is a subgroup of a group G , a coset of H in G is the set $\{gh \mid h \in H\}$ for some $g \in G$. (See the notes on group theory on the course web page if you need further background with groups.)

- (10 pts.) Let M and N be two elements of $N(S)$. Recall that this implies that M and N map codewords to codewords, meaning they are unitaries acting on the logical qubit(s) of the code. Show that $M|\psi\rangle = N|\psi\rangle$ for all codewords $|\psi\rangle$ iff M and N are in the same coset of S in $N(S)$. Therefore, each coset of S in $N(S)$ represents a different logical unitary.
- (10 pts.) Show that two Pauli errors E and F have the same error syndrome for a stabilizer code S iff they are in the same coset of $N(S)$ in \mathcal{P}_n . Thus, the error syndrome is a property of cosets of $N(S)$.
- (10 pts.) Suppose that a Pauli error E occurs on a codeword of the stabilizer code S , but we try to correct it by instead applying Pauli error F with the same error syndrome as E . Show that the resulting state is always a codeword.