Problem Set #8

CMSC 657
Instructor: Daniel Gottesman

Due on Gradescope, Thursday, Oct. 24, 2024, at 5:00 PM

Remember to mention any other students you worked with, as well as any outside resources (including AI tools) and how you used them.

Problem #1. A New Quantum Computer Implementation (60 pts.)

A new particle, the zanyon, has just been discovered! In this problem, you will investigate how to build a quantum computer out of zanyons.

Each zanyon has two properties, funness and weirdness. A zanyon's funness can be $|\text{fun}\rangle$, $|\text{boring}\rangle$, or any superposition of $|\text{fun}\rangle$ and $|\text{boring}\rangle$. A zanyon's weirdness can be $|\text{normal}\rangle$, $|\text{bizarre}\rangle$, or any superposition of the two.

Experimentalists have learned have to move zanyons around however they like. Funness can be measured directly when a zanyon interacts with normal matter, in the basis $\{|\text{fun}\rangle, |\text{boring}\rangle\}$. Weirdness can only be measured by observing a zanyon's effects on other zanyons. Zanyons are hard spheres, so cannot be placed in exactly the same place, but if two zanyons are touched together, they undergo the following interaction:

1. If both zanyons are $|\text{boring}\rangle$, nothing happens.
2. If one zanyon is $|\text{boring}\rangle$, and the other zanyon is $|\text{fun}\rangle$ and $|\text{normal}\rangle$, nothing happens.
3. If one zanyon is $|\text{boring}\rangle$, and the other zanyon is $|\text{fun}\rangle$ and $|\text{bizarre}\rangle$, the boring zanyon's weirdness quantum number experiences the Hamiltonian $ir(|\text{normal}\rangle\langle\text{bizarre}| - |\text{bizarre}\rangle\langle\text{normal}|)$.
4. If both zanyons are $|\text{fun}\rangle$, they experience the Hamiltonian $p\,|\text{bizarre}\rangle \otimes |\text{bizarre}\rangle\langle\text{bizarre}| \otimes \langle\text{bizarre}|$.

The constants r and p are experimentally measured quantities which determine how strongly zanyons interact. If the zanyons are not touching each other, their weirdnesses do not change.

New zanyons can be created at will using a secret recipe involving chocolate, balloons, and liquid nitrogen. When a new zanyon is created, it is always in the state $|\text{fun}\rangle \otimes |\text{bizarre}\rangle$. In addition, experiments have discovered that magnetic fields change a zanyon's funness. In a field of strength B, a zanyon's funness quantum number experiences the Hamiltonian $iB(|\text{fun}\rangle\langle\text{boring}| - |\text{boring}\rangle\langle\text{fun}|)$.

a) (10 pts.) Explain how to put a newly created zanyon into a state of funness $\cos\theta|\text{fun}\rangle + \sin\theta|\text{boring}\rangle$ for arbitrary θ.

b) (5 pts.) Suppose we are given a zanyon with an unknown state of funness. Explain how to put it in a $|\text{fun}\rangle$ state.

c) (5 pts.) When we build a quantum computer from zanyons, we will store the quantum information in the weirdness degree of freedom, with $|0\rangle = |\text{normal}\rangle$ and $|1\rangle = |\text{bizarre}\rangle$. Why is the weirdness quantum number superior to the funness quantum number for storing quantum information?

d) (10 pts.) Explain how to do a CNOT gate between two zanyons.

e) (10 pts.) Explain how to do a universal set of gates on zanyons.

f) (10 pts.) How can we measure the weirdness of a zanyon?

Hint: Put the zanyon you want to measure in the funness state $|\text{fun}\rangle + |\text{boring}\rangle$.

g) (5 pts.) What do you expect to be the major sources of error in our zanyon quantum computer?

h) (5 pts.) Comment on how well our zanyon quantum computer meets the first five DiVincenzo criteria.