Problem Set #7

CMSC 657
Instructor: Daniel Gottesman

Due on Gradescope, Thursday, Oct. 17, 2024, at 5:00 PM

Remember to mention any other students you worked with, as well as any outside resources (including AI tools) and how you used them.

Problem #1. Grover's algorithm for Monte Carlo (20 pts.)

Suppose we want to perform a Monte Carlo algorithm to determine the volume V of a region R in d dimensions contained in the unit cube. The classical Monte Carlo algorithm picks random points and calculates what fraction f of those points lie within R using an algorithm $A(x)$ that decides if the point $x \in R$. f then gives an estimate of V.

Suppose we know only that V is either exactly 0 or at least ϵ, but otherwise we have no idea what V is. We wish to find out if V is 0 or not. The classical algorithm uses $\Theta(1/\epsilon)$ points on average to find a non-zero volume in the worst case.

Using Grover's algorithm, show that a quantum computer can determine with high probability whether R has non-zero volume using only $O(1/\sqrt{\epsilon})$ evaluations of $A(x)$. (Remember to specify which variant of Grover's algorithm you are using — that is, whether the number of marked elements is known and how many marked elements there are.)

Problem #2. Quantum Simulation (40 pts.)

In this problem, X_i and Z_i refer to the Pauli matrices X and Z acting on the ith qubit, tensored with the identity on the other qubits.

a) (10 pts.) Suppose we have two qubits and $H = Z_1 \otimes Z_2$. Find an explicit 4×4 matrix $U_{ZZ}(t)$ for $e^{-iHt/\hbar}$.

b) (10 pts.) Now suppose we have three qubits and $H = a(Z_1 \otimes Z_2 + Z_2 \otimes Z_3)$. Write a circuit to simulate this Hamiltonian up to time t. You may use arbitrary single-qubit gates, CNOT gates, and two-qubit gates in the family $U_{ZZ}(t)$ from part a.

c) (10 pts.) Again we have three qubits and $H = b(X_1 + X_2 + X_3)$. Write a circuit to simulate this Hamiltonian up to time t. You may use arbitrary single-qubit gates, CNOT gates, and two-qubit gates in the family $U_{ZZ}(t)$ from part a.

d) (10 pts.) Now, still with three qubits, let $H = a(Z_1 \otimes Z_2 + Z_2 \otimes Z_3) + b(X_1 + X_2 + X_3)$. Use the results of parts b and c to give a circuit to simulate this Hamiltonian up to a time $T = 1$ using four time steps ($\Delta t = 1/4$) by breaking the Hamiltonian up into two pieces. Again, you may use arbitrary single-qubit gates, CNOT gates, and two-qubit gates in the family $U_{ZZ}(t)$ from part a.