Problem Set #6

CMSC 657
Instructor: Daniel Gottesman

Due on Gradescope, Thursday, Oct. 10, 2024, at 5:00 PM

Remember to mention any other students you worked with, as well as any outside resources (including AI tools) and how you used them.

Problem #1. The Qutrit Quantum Fourier Transform (60 pts.)

For this problem, we will consider circuits involving *qutrits*, 3-dimensional Hilbert spaces. If we have n qutrits, we therefore have a Hilbert space of dimension 3^n.

a) (30 pts.) Find a circuit to implement the Fourier transform mod 3^n:

$$\mathcal{F}_{3^n}|a\rangle = \frac{1}{\sqrt{3^n}} \sum_{b=0}^{3^n-1} \omega^{ab}|b\rangle, \tag{1}$$

where $\omega = \exp(2\pi i/3^n)$. Express your circuit in terms of quantum gates acting on one and two qu*trits*, not in terms of qubit gates.

b) (30 pts.) For the following problem, find a quantum algorithm that uses only one quantum oracle query: The classical version of the oracle takes as input a string of trits $x \in \mathbb{Z}_3^n$ and outputs $x \cdot y + z$ for some $y \in \mathbb{Z}_3^n$. The dot product is taken considering x and y as vectors in the n-dimensional vector space over $\mathbb{Z}_3$. That is, addition and multiplication are taken mod 3. The quantum version of this oracle takes the inputs $|x\rangle|a\rangle$ and outputs $|x\rangle|a + (x \cdot y + z)\rangle$. The goal is to find y.