Problem Set #5

CMSC 657
Instructor: Daniel Gottesman

Due on Gradescope, Thursday, Oct. 3, 2024, at 5:00 PM

Remember to mention any other students you worked with, as well as any outside resources (including AI tools) and how you used them.

Problem #1. Parity Oracle (60 pts.)

For this problem, consider the following family of oracles: $O_a(x) = x \cdot a$. Here a is an unknown property of the oracle and x is the input. a and x are both n-bit strings, which we can consider as binary vectors of length n (vectors with 0/1 components), and $x \cdot a$ is the binary dot product, giving a bit as output. The goal is to find a.

a) (10 pts.) Find a single classical query (i.e., value of x to input into the oracle) that will tell you the ith bit of a.

b) (10 pts.) Find a classical query algorithm that will find the full value of a using n queries.

c) (10 pts.) Do you think that there is a randomized classical algorithm to find a using $o(n)$ queries? Remember that a classical query algorithm can use any input, not just the ones you used in part b. (You do not need a complete proof, but explain your reasoning.)

d) (10 pts.) Find $H^{\otimes n}|y\rangle$ when y is a length n bit string. (That is, Hadamard applied to each of the n qubits.)

e) (10 pts.) Write down the action of the quantum oracle that corresponds to O_a.

f) (10 pts.) Find a quantum algorithm to determine a using a constant number of queries. Full points if you can do it using just 1 query. Hint: You may want to use the results of part e.