Problem Set #4

CMSC 657
Instructor: Daniel Gottesman

Due on Gradescope, Thursday, Sep. 26, 2024, at 5:00 PM

Remember to mention any other students you worked with, as well as any outside resources (including AI tools) and how you used them.

Problem #1. Entanglement Swapping (60 pts.)

Alice and Bob would like to share the entangled state $|\Phi^+\rangle = |00\rangle + |11\rangle$. Unfortunately, they do not initially share any entanglement, but fortunately, they have a mutual friend, Charlie. Alice shares a copy of $|\Phi^+\rangle$ with Charlie, and Bob also shares a copy of $|\Phi^+\rangle$ with Charlie.

a) (10 pts.) Write down the initial state in the standard basis ($|0\rangle$, $|1\rangle$), with the first qubit belonging to Alice, the second and third qubits belonging to Charlie, and the fourth qubit belonging to Bob.

b) (25 pts.) Suppose Charlie performs a Bell measurement on his two qubits (one of which is entangled with Alice and the other of which is entangled with Bob). For each possible measurement outcome, give the probability with which it occurs and the resulting post-measurement state for Alice and Bob.

c) (25 pts.) Describe a protocol whereby Charlie sends a classical message to Alice, and Alice processes her quantum state, such that Alice and Bob share the state $|\Phi^+\rangle$ at the end of the protocol. Draw the quantum circuit corresponding to this protocol.