Problem Set #3

CMSC 657
Instructor: Daniel Gottesman

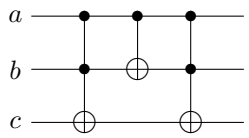
Due on Gradescope, Thursday, Sep. 19, 2024, at 5:00 PM

Remember to mention any other students you worked with, as well as any outside resources (including AI tools) and how you used them.

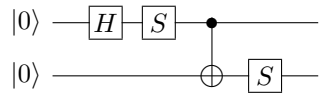
Problem #1. Practice Evaluating Circuits (20 pts.)

Compute the outcome state for the following circuits. You can use any convenient method (not necessarily bra-ket notation).

a) (10 pts.) This is a classical circuit.



b) (10 pts.) In this part, $S = e^{i\pi/4}R_{\pi/4} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$ and H is the Hadamard transform.



Problem #2. Finding Circuits (20 pts.)

a) (10 pts.) Find a quantum circuit to create the state $\sqrt{\frac{1}{3}}\,|00\rangle + \sqrt{\frac{2}{3}}\,|11\rangle$ starting with two qubits in the state $|0\rangle$. Do not use any extra ancilla qubits. You can use any 1-qubit gates and the CNOT gate.

b) (10 pts.) Find a quantum circuit to perform the 3-qubit projective measurement given by $\Pi_0 = |111\rangle\langle 111|$, $\Pi_1 = I - \Pi_0$ on an arbitrary input state. You may use ancilla qubits which start in the state $|0\rangle$, any 1-qubit gates, the CNOT gate, the Toffoli gate, and measurement in the $\{|0\rangle, |1\rangle\}$ basis. Make sure you are not collapsing the state more than is necessary for the specified measurement.

Problem #3. Circuit Identity (20 pts.)

Prove that the following circuit identity holds (i.e., that it is true for all inputs). H is the Hadamard transform and $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.