Problem Set #2

CMSC 657
Instructor: Daniel Gottesman

Due on Gradescope, Thursday, Sep. 12, 2024, at 5:00 PM

Remember to mention any other students you worked with, as well as any outside resources (including AI tools) and how you used them.

Problem #1. Practice With Bras and Kets and Density Matrices (24 pts.)

Calculate the following using the bra-ket notation. For this problem, let $U = \cos\theta\,|0\rangle\langle 0| - \sin\theta\,|0\rangle\langle 1| + \sin\theta\,|1\rangle\langle 0| + \cos\theta\,|1\rangle\langle 1|$. For full credit, you **must** use bra-ket notation.

a) (6 pts.) Consider the 2-qubit unnormalized pure state $|01\rangle + 2|10\rangle - |11\rangle$. Calculate the density matrix of the first qubit. Remember to normalize the state.

b) (6 pts.) When $\rho = \frac{2}{3}|0\rangle\langle 0| + \frac{1}{3}|1\rangle\langle 1|$, calculate the resulting state if U is applied to ρ.

c) (6 pts.) Consider the CPTP map $\mathcal{C}$ with Kraus operators $A_0 = \sqrt{1-p}(|0\rangle\langle 0| + |1\rangle\langle 1|)$, $A_1 = \sqrt{p}(|0\rangle\langle 0| - |1\rangle\langle 1|)$ and let $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Calculate $\mathcal{C}(|\psi\rangle\langle\psi|)$. **Note:** Fixed from $1-p$ and p.

d) (6 pts.) Consider the POVM with three effects $M_0 = \frac{2}{3}|0\rangle\langle 0|$, $M_1 = |\psi\rangle\langle\psi|$, with $|\psi\rangle = -\frac{1}{\sqrt{6}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$, and $M_2 = |\phi\rangle\langle\phi|$. Find $|\phi\rangle$.

Problem #2. Partial Measurements (36 pts.)

For this problem, assume we have a composite system $A \otimes B$ and a state ρ (possibly mixed, possibly entangled).

a) (18 pts.) Prove that a partial measurement in basis $|i\rangle_A$ on A has the same distribution of outcomes i as the measurement in basis $|i\rangle_A$ on the state $\mathrm{Tr}_B\,\rho$. That is, discarding the system B doesn't change the outcome of measurements on A.

b) (18 pts.) Prove that partial measurement on ρ in basis $|i\rangle_A$ on A followed by partial measurement in basis $|j\rangle_B$ on B has the same probability distribution on outcomes (i, j) as a complete measurement on ρ in basis $|ij\rangle_{AB}$.