Problem Set #1

CMSC 657
Instructor: Daniel Gottesman

Due on Gradescope, Thursday, Sep. 5, 2024, at 5:00 PM

Remember to mention any other students you worked with, as well as any outside resources (including AI tools) and how you used them.

Problem #1. Practice With Bras and Kets (60 pts.)

Calculate the following using the bra-ket notation. For this problem, let $U = \cos\theta\,|0\rangle\langle 0| - \sin\theta\,|0\rangle\langle 1| + \sin\theta\,|1\rangle\langle 0| + \cos\theta\,|1\rangle\langle 1|$. Note that the states in parts a and b involve complex numbers. For full credit, you **must** use bra-ket notation.

a) (6 pts.) Calculate the inner product of $\frac{1}{\sqrt{3}}(|00\rangle + |01\rangle - |11\rangle)$ and $\frac{1}{2}|00\rangle + \frac{i}{2}|01\rangle - \frac{i}{\sqrt{2}}|10\rangle$.

b) (6 pts.) Find the probability of measuring 1 if we measure the 0/1 basis for the normalized version of $i|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$.

c) (6 pts.) Calculate $U^\dagger U$.

d) (6 pts.) Let $|\psi\rangle = |0\rangle + |1\rangle$. Find $U|\psi\rangle$.

e) (6 pts.) Let $|\phi\rangle = |00\rangle + |11\rangle$. Find $(U \otimes I)|\phi\rangle$.

f) (6 pts.) Again let $|\phi\rangle = |00\rangle + |11\rangle$. Find $(U \otimes U)|\phi\rangle$.

g) (12 pts.) Find V such that $(I \otimes V)|\phi\rangle = (U \otimes I)|\phi\rangle$, where $|\phi\rangle = |00\rangle + |11\rangle$ again.

h) (6 pts.) Let $\Pi_1 = |+\rangle\langle +|\otimes I$ and $\Pi_2 = |-\rangle\langle -|\otimes I$, where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$. Calculate the probabilities of both outcomes of the projective measurement $\{\Pi_1, \Pi_2\}$ on the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

i) (6 pts.) For the same Π_1 and Π_2 as in part g, find the residual state if you make this measurement on the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and get the outcome 1 (that is, Π_1).