CMSC 657: Introduction to Quantum Information Processing
Lecture 17

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1 Superconducting Quantum Computers

1.1 Superconductors

In some materials, at very low temperatures, electrical resistance disappears and the materials become superconductors. In a regular conductor, some electrons are not tightly bound to the atoms and can instead move easily from atom to atom, allowing current to flow (since current is movement of charge). However, this process is not totally regular. Occasionally, each electron will hit an atom and have its movement disrupted. Some energy is lost to heat, and the electron slows down, reducing the current. This produces resistance.

In a superconductor, the electrons pair up into *Cooper pairs*. A Cooper pair is a boson, not a fermion, which means that multiple Cooper pairs can be in the same state; indeed, it is statistically favorable for them to be in the same state. At low temperatures, many Cooper pairs will enter the same state, a *superconducting condensate*. The effective Hamiltonian of the electrons has a *spectral gap*: between the ground state (Cooper pair) and the next highest energy level (electrons going their separate ways), there is some space that stays constant as the size of material increases. Scattering an electron off an atom now takes energy, and at low temperatures that energy is not available, so the Cooper pairs just keep moving, leading to 0 resistance. Remarkably, this is actually *zero* resistance, not just small. A current in a superconductor can persist indefinitely.

To make superconductors useful for quantum computing, we need some more detailed control of the behavior of the superconductor. This is provided by *Josephson junctions*. A Josephson junction is a connection between two pieces of superconductor with a small gap, vacuum or an insulator, between them. Cooper pairs can still pass through the gap through quantum *tunneling*: Remember that for quantum particles, momentum is $i\hbar\frac{\partial}{\partial x}$, so a sharp change in the wavefunction has a high energy cost. Therefore, when a quantum particle reaches a region where it would be classically forbidden (for instance, high potential energy in that region), the wavefunction doesn't stop abruptly, but dies off exponentially. If the region is very small, as with the gap in a Josephson junction, there will still be non-negligible quantum amplitude of the particle on the other side of region, and the particle can continue moving normally there. This is quantum tunneling. The rate of the exponential decay depends on the difference between the particle's energy and the energy in the region, and in a Josephson junction, we can adjust this by changing the voltage across the gap.

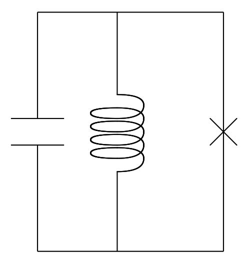
When all is said and done, the Josephson junction in a quantum circuit is going to act like a non-linear inductor. When the current changes, an inductor creates a voltage opposing the change. A normal, non-superconducting inductor is basically linear and the voltage is directly proportional to the derivative of the current. In a Josephson junction, the dependence is non-linear.

1.2 Superconducting circuits and qubits

There are a wide variety of superconducting qubits that people have designed, but they all share some commonalities. They all involve circuits with Josephson junctions, inductors, and capacitors. Since these are superconductors, there are no resistors.

For relatively large superconductors, there are many Cooper pairs and the behavior is basically classical in respects other than superconductivity. But for smaller superconductors, the collective behavior results in just a single effective degree of freedom, movement of all the Cooper pairs together, and we can get superpositions, making this a candidate for a qubit. These are *mesoscopic* systems of intermediate size between atomic and macroscopic, classical objects. There are still many Cooper pairs, unlike with ion traps where the qubit is one ion or one electron. This is possible because the collective state of a superconductor is well isolated from the outside world. Nevertheless, decoherence rates in superconductors are much shorter than in ion traps, tens of microseconds rather than milliseconds. A common cause of decoherence is impurities, other kinds of atoms nearby that interact a bit with the qubit. Since the time to do a gate is much faster than in an ion trap (10s of nanoseconds), the error rates can be in the same ballpark (although not quite as good) as for ion traps. The main advantage is that since they are solid-state systems, they are easier to scale up by simply manufacturing more of them.

There are a number of different companies making superconducting quantum computers. Currently Google has a 105-qubit device and IBM's largest publicly accessible device has 156 qubits (but they have announced a device with over 1000 qubits). Error rates for 2-qubit gates are usually a few percent, a bit higher than for ion traps.



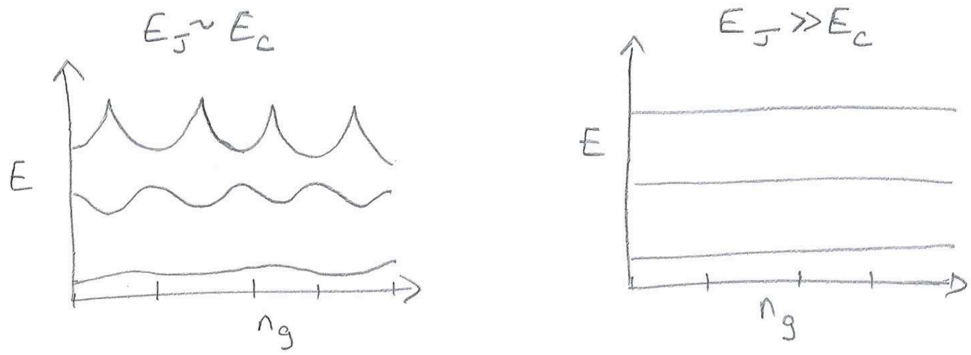
In this schematic circuit representation of a superconducting qubit, the parallel lines represent a capacitor, the swirly lines represent a regular linear inductor, and the X is the Josephson junction.

The Hamiltonian in such a circuit is of the form

$$H = E_C(\hat{n} - n_g)^2 + E_L(\hat{\phi} - \phi_e)^2/2 - E_J \cos\hat{\phi}. \tag{1}$$

The variables are $\hat{n}$, the number of Cooper pairs on the capacitor, and $\hat{\phi}$, the phase change in the superconducting wave function over the inductor. They are conjugate variables, meaning they have a non-trivial commutation relationship like position and momentum. Thus, there are no simultaneous eigenstates of $\hat{n}$ and $\hat{\phi}$. E_C, E_L, and E_J are determined by the design of the qubit and are fixed once the qubit is fabricated. n_g and ϕ_e are controllable. n_g is determined by the voltage placed on the capacitor and ϕ_e is a phase contribution which is determined by the magnetic field.

The different types of superconducting qubits have different detailed circuit arrangements with different combinations of these elements and also differ in the parameters that appear in the Hamiltonian. If we just had the first two terms, it would be a harmonic oscillator. The third term makes it an *anharmonic oscillator*.



The qubit is basically then a pair of these energy levels. Cranking up the capacitance makes E_C small which leads to flat energy levels as a function of n_g. This is good, as it means the system is less sensitive to charge fluctuations on the capacitor. However, that makes control more difficult, since the oscillator becomes more harmonic, making it more difficult to pick out the desired pair of energy levels. In the *transmon* qubit design, we use an intermediate regime where the energy levels are pretty flat with n_g but there is still some anharmonicity.

(Why does it look this way? When E_J is large, the last term dominates, so ϕ (the expectation value of $\hat{\phi}$) should be very close to a maximum, e.g., $\phi = 0$. Then we can approximate $\cos\phi = 1 - \phi^2/2$, and the Hamiltonian becomes a harmonic oscillator again. If E_C is large, the first term dominates, meaning we want n to be very close to its minimum. Then ϕ is very uncertain, so the higher-order terms in the power series of $\cos\phi$ are also important.)

1.3 Superconducting gates

There are a number of options of how to do gates, leading to many variations of design even among transmons.

On option is to include a Cooper pair box to which we can add magnetic flux. This means the superconducting phase must go up to compensate, which has the effect of changing the energy splitting Ω of the qubit. We can apply phase shifts by turning this shift on and off. This also can bring the circuit in and out of resonance with a resonator or between two qubits.

Another option is to couple the superconducting circuit to a microwave resonator, giving an effective Jaynes-Cummings Hamiltonian

$$H \approx \hbar\omega a^\dagger a + \hbar\Omega/2Z + \hbar g(a^\dagger \sigma^- + a\sigma^+). \tag{2}$$

Here a is the annihiliation operator for a photon in the resonator, and $\sigma^\pm = (X \pm iY)/2$ are "raising" and "lowering" operators for the qubit. This is basically the same Hamiltonian we had coupling the ions to the phonon mode, but now the coupling g is not going to be our main control parameter because it tends to be hard to change. The typical parameters are also different.

With the coupling always on, one option is to go for strong coupling; then the eigenstates become entangled between the "qubit" and the resonator. These are our actual qubit states. If the cavity is off-resonant compared to the superconducting circuit, the entanglement will not be large, but it is there.

If we drive the resonator by adding strong microwave fields, the resonator get pushed towards a classical state and the entanglement with the qubit ensures that the actual classical state will be correlated with the qubit state. This lets us do a measurement.

A weaker microwave drive uses the entanglement between cavity and circuit to induce bit flips, a σ^x Hamiltonian. Microwave pulses can do X or Y rotations.

For two-qubit gates, again there are many options. One is to couple two circuits via a wire and a capacitor. This produces an approximate coupling Hamiltonian, when the qubits have the same energy, of the form

$$H = \sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+. \tag{3}$$

This Hamiltonian has no effect on the $|00\rangle$ and $|11\rangle$ states, but it will rotate between the $|01\rangle$ and $|10\rangle$ states. If run for the right amount of time, it swaps the states of the two qubits. But: if run for a fraction of that time, this is a non-trivial entangling gate. E.g., the square root of SWAP is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

It turns out that this plus single-qubit gates also forms a universal gate set. There are other options for two-qubit gates that more directly produce a controlled-phase gate or a CNOT.

2 Other Platforms

Two other leading systems for building quantum computers are photons and cold neutral atoms in optical lattices. There are other systems under investigation, but they are not comparably advanced to these or ion traps or superconductors.

Photons are individual particles of light and are really great for quantum communication, as they can preserve quantum states over long distances and are easy to produce, but it is hard to keep them around in one place and more importantly, they interact with each other only weakly. There are various strategies for dealing with this: You can have a hybrid matter-light system using the matter to produce a strong interaction, or you can take advantage of measurement non-linearities to produce effective interactions between the photons. Companies building photonic quantum computers include PsiQuantum and Xanadu.

Cold atoms use optical lattices to trap neutral atoms and cool them. An optical lattice is composed of criss-crossing laser beams, which form standing wave patterns with nodes (points of weak field) and antinodes (strong fields). The lasers interact with the atoms, and under the right conditions, the atoms spread out into a lattice matching the standing wave pattern. They can be moved around using additional lasers or by manipulating the trapping lasers and can be made to interact. A company building cold atom quantum computers is QuEra.