CMSC 657: Introduction to Quantum Information Processing
Lecture 16

Instructor: Daniel Gottesman

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1 Ion Traps, Continued

1.1 Two-qubit ion traps and gates

Recall that the movement of a single ion in a linear trap is a harmonic oscillator, with Hamiltonian $H = \hat{p}^2/2m + \frac{1}{2}m\nu^2\hat{x}^2$.

The classical harmonic oscillator vibrates according to a sine function with period ν. If you solve Schrödinger's equation for a quantum harmonic oscillator, you get an infinite set of equally spaced energy levels $|0\rangle, |1\rangle, |2\rangle, \ldots$ with energies $\hbar\nu(n+\frac{1}{2})$. We like to write down *creation* and *annihilation* operators $a^\dagger$ and a. They have the effect of moving the state up or down an energy level:

$$a|n\rangle = \sqrt{n}|n-1\rangle \quad (n > 0) \tag{1}$$
$$a|0\rangle = 0 \tag{2}$$
$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle. \tag{3}$$

The a and $a^\dagger$ are not unitary. Instead

$$a^\dagger a|n\rangle = n|n\rangle \tag{4}$$

is a *number operator* that returns the number of the eigenstate. We like to think of n as representing the number of quantum excitations in the harmonic oscillator, so $a^\dagger$ and a create and destroy these quanta; thus their names. We can rewrite the Hamiltonian of the harmonic oscillator in terms of the creation and annihilation operators

$$H = \hbar\nu a^\dagger a + \frac{1}{2}\hbar\nu. \tag{5}$$

Here, ν is the frequency of the oscillator.

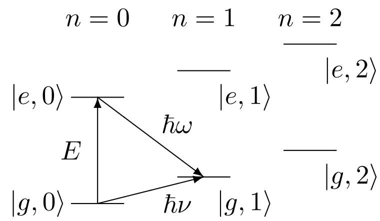
For the phonon modes of the ion trap, there are many atoms, so the solutions are more complicated than for a single simple harmonic oscillator. Each mode of motion is separately an almost harmonic oscillator and can be described as above. The different modes have different frequencies and can interact separately with the atoms.

Let us focus on the center of mass mode. It is an approximately harmonic oscillator, at least for low-amplitude oscillations, with Hamiltonian

$$H_{\text{phonon}} = \hbar\nu a^\dagger a, \tag{6}$$

as discussed above.

We can then look at the joint energy levels of phonon plus one atom, the Hamiltonian $H_{\text{atom}} + H_{\text{phonon}}$. At the moment, the electron and the phonon do not interact. Their states form a ladder of joint states $|e, n\rangle$ and $|g, n\rangle$.



Let

$$\hbar\omega = E - \hbar\nu. \tag{7}$$

When we shine a laser with frequency ω, we get an interaction Hamiltonian of the form

$$H_{\text{laser}} = J(e^{i\omega t}|g\rangle\langle e| \otimes a^\dagger + e^{-i\omega t}|e\rangle\langle g| \otimes a). \tag{8}$$

It couples the energy levels $|e, n\rangle$ and $|g, n+1\rangle$. As before, if we turn the laser on for some time, it induces an oscillation between these two levels. But note that $|g, 0\rangle$ does not couple to anything. This idea is behind the two-qubit Cirac-Zoller gate.

In particular, we will use the phonon mode and an additional energy level $|e'\rangle$ with energy E'. It is not unstable like $|f\rangle$, but does not need to be as stable as $|e\rangle$ and $|g\rangle$ since we are not staying in it for long. Let $\omega' = (E' - E)/\hbar - \nu$.

At the start of the gate, the phonon mode is $|0\rangle_p$, the vacuum state. I will use subscripts 1 and 2 for the ions 1 and 2 and subscript p for the phonon mode. We do the following pulse sequence:

1. Do a $\pi/2$ pulse on ion 1 with frequency ω. This takes $|e\rangle_1|0\rangle_p \mapsto |g\rangle_1|1\rangle_p$, while $|g\rangle_1|0\rangle_p \mapsto |g\rangle_1|0\rangle_p$.

2. Do a π pulse on ion 2 with frequency ω'. Since ω' is only resonant with the transitions $|e\rangle_2|n\rangle_p$ to $|e'\rangle_2|n-1\rangle_p$, this laser does nothing if the state is $|e\rangle_2|0\rangle_p$, $|g\rangle_2|0\rangle_p$, or $|g\rangle_2|1\rangle_p$, but if the state is $|e\rangle_2|1\rangle_p$, it acquires a phase -1 (which is no longer a global phase, since it is relative to those other states that did not get the phase).

3. Undo step 1 with a $-\pi/2$ pulse, again with frequency ω.

The last step always returns the phonon to $|0\rangle_p$ (in the absence of error). In the meantime, nothing has happened in any step if the first ion is $|g\rangle_1$ and nothing happens in step 2 if the second ion is $|g\rangle_2$ (which means that the first and third steps cancel with no overall effect). The only time anything changes is if both ions are excited $|e\rangle_1|e\rangle_2$, in which case we have a phase -1.

Thus, we get a controlled-phase gate $C - Z$:

$$|g\rangle_1|g\rangle_2 \mapsto |g\rangle_1|g\rangle_2 \tag{9}$$
$$|g\rangle_1|e\rangle_2 \mapsto |g\rangle_1|e\rangle_2 \tag{10}$$
$$|e\rangle_1|g\rangle_2 \mapsto |e\rangle_1|g\rangle_2 \tag{11}$$
$$|e\rangle_1|e\rangle_2 \mapsto -|e\rangle_1|e\rangle_2. \tag{12}$$

This, along with single-qubit gates, is universal (condition 4). You can get a $CNOT$ from it using Hadamards:

$$CNOT = (I \otimes H)C - Z(I \otimes H). \tag{13}$$

Note that initialization is more challenging for the two-qubit ion trap. Note only do we need to get the atoms into their ground states, but we need the phonon in the vacuum mode. This can be done via "sideband cooling": If we put all atoms in $|g\rangle$ and do a $\pi/2$ pulse at frequency ω, the state goes from $|g\rangle_1|n\rangle_p$ to $|e\rangle_1|n-1\rangle_p$. Then we can reset atom 1 to $|g\rangle$ and repeat, sucking all of the phonons out of the mode.

1.2 Errors

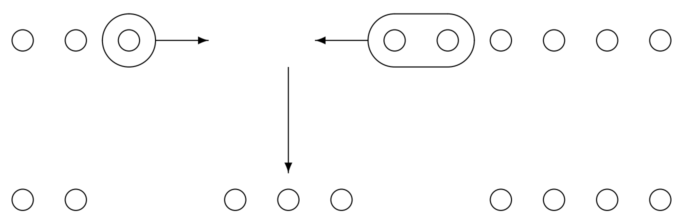
For ion traps, the storage and single-qubit gates are quite robust. There can be errors from spontaneous emission of the $|e\rangle$ state into the $|g\rangle$ state, but the energy levels selected for qubits are chosen so that this is quite rare. More important are timing or frequency fluctuations in the lasers controlling the qubits, and stray magnetic fields can shift the energy levels, causing phase errors.

The two-qubit gates have a higher error rate, about 0.1% currently in the best ion trap quantum computers. There are the same issues as with single-qubit gates, but also spontaneous emission from the $|e'\rangle$ state can cause errors. The biggest source, however, is usually problems with the phonon mode. The Cirac-Zoller gate is particularly susceptible to this since it requires the requires the phonon to start in exactly the $|0\rangle$ (vacuum) state, which is challenging. Today people mostly use other gate methods which are a bit more robust, but it is still the case that the phonon is a major source of errors.

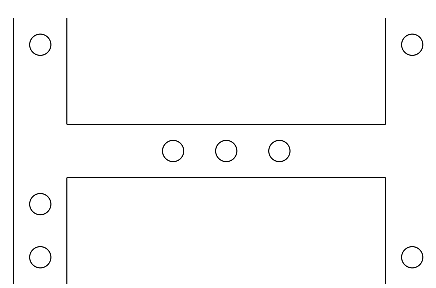
1.3 Scaling up

There is a limit to how many ions you can put in one trap. As you put more and more ions in the trap, the phonon modes start to crowd together if you make the trap bigger and the confinement weaker or the ions get too close to each other and it becomes difficult to illuminate them one-by-one. Therefore, we will need some way of combining different ion traps to make a big quantum computer. There are two basic ideas.

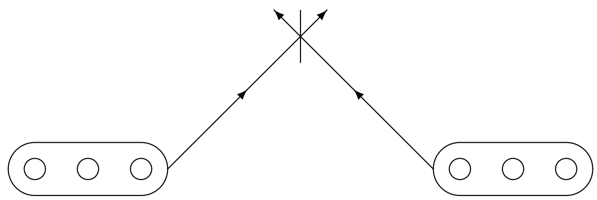
The first is to physically move the ions around. The leading devices today do this by trapping many ions using the same trap but separating them. They can be moved together into an interaction region in the middle of the trap by manipulating the trapping fields.



This allows more ions in a trap with the ability to still interact in controllable groups, but eventually it will still hit a limit. One option to scale up further is to store the ions in separate but linked trapping regions and bringing them together to interact.



A second strategy is to link separate ion traps with light. Put the ion trap in an optical cavity, basically a tiny box with extremely good mirrors that cause light to stay trapped as well for relatively long periods of time (for light). The box has a size that is a multiple of a particular optical wavelength that interacts well with the atom. Individual particles of light are called photons and they have a mode in the optical cavity with basically the same properties as the phonon mode before. This photon mode can interact with the atom, but if we pick an optical cavity whose mirrors are not perfect, occasionally the light can leak out. Take two such traps and put the leaking photon modes through a beam splitter. This gives us an EPR pair between the photon modes of two cavities. This can be interfaced with the ions trapped in the cavities to make an EPR pair between ions. Then use teleportation.



Ion traps with a single atom have decoherence times on the order of tens of seconds, but for a quantum computer, there are other dominant decoherence mechanisms due to the gate operations. The largest ion trap computers today are on the order of 10s of qubits. Error rates have been shown as low as about 10^{-3} for a two-qubit gate.