Lecture on October 24th, 2017
Date: October 27, 2017

Summary of Lecture 17

Reading: [Arora-Barak (AB)] 21.2

- We apply random walks over expander graphs to achieve randomness-efficient error reductions.
- Note that in the lecture, we only deal with one-sided error class RP. For RP, we can define the set of "bad" seeds, which are seeds that will make the algorithm reject even when the input is in the language. When the input is not in the language, then we will always reject because of the one-sided error.
- The following high-level strategy follows from Chap 21.2 of [AB]. We will relate the probability of hitting all bad seeds during a random-walk over an expander graph to the ℓ_2 norm of the vector $(BA)^{k-1}B\vec{1}$. And then we analyze such ℓ_2 norm and make use of the properties of spectral norm of matrices.
- Important properties include: (1) $\|A\| \leq 1$ for a stochastic matrix A. (2) $\|AB\| \leq \|A\|\|B\|$.
- The expander graph comes into play because we have a decomposition of A for (n, d, λ) expander G as follows:

  $$A = (1 - \lambda)J + \lambda C,$$

  where J is the all-one matrix normalized by $\frac{1}{n}$ (i.e., the matrix for the n-clique graph) and C is any matrix such that $\|C\| \leq 1$. We didn't go through the proof, which is in Chap 21.2 of [AB].