Summary of Lecture 14

**Reading:** [Arora-Barak (AB)] 7.1, 7.2, 7.3, 7.4

- Randomized algorithms for **polynomial identity test (PIT)** and why it works.

- Two ways to introduce randomness into Turing machines: (1) a random tape; (2) a random choice of the transition function.

- The definition of RP, CoRP (one-sided error), BPP (two-sided error), and ZPP (zero-error). Note that we could have two equivalent definitions of ZPP: (proof by the Markov inequality)
  
  - A probabilistic TM M that always output the correct answer and the expected running time of M is polynomial.
  
  - A probabilistic TM M that runs in polynomial time, and outputs either a correct answer (with probability at least $\frac{1}{2}$) or "I don’t know".

- Error-reduction for BPP using the Chernoff bound.