Summary of Lecture 10

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Reading: [Arora-Barak (AB)] Chap 5.1, 5.2, 5.3.

- Motivating examples and definitions of Σ_i, Π_i and the polynomial hierarchy PH.

- If $\Sigma_i = \Pi_i$, then PH collapses to Σ_i. The proof makes use of two different quantified boolean formulas representing the same language (from the assumption), and reduces the number of quantifiers by combining adjacent quantifiers of the same type.

- PH is strictly contained in PSPACE unless PH collapses to Σ_{i^*} for some i^*. This is because PSPACE has a complete problem, while there is no complete problem for PH unless PH collapses to Σ_{i^*} for some i^*. One can use a property like this to show two complexity classes are different.

  Here is an another example (optional): try to show SPACE(n) $\neq$ NP.

- Alternating Turing Machines (ATMs).