Lecture on September 26th, 2017
Date: September 30, 2017

Summary of Lecture 9

Reading: [Arora-Barak (AB)] Chap 4.3.1, 4.3.2.

- Certificate definition of NL. [Definition 4.19]
- NL=coNL. [Theorem 4.20].

1 Equivalence between two definitions of NL

Here we sketch about the two definitions of NL and why they are equivalent to each other. Two definitions are

(I) NL defined as Non-deterministic TMs with $O(\log(n))$ size work tape.

(II) NL defined as TMs with $O(\log(n))$ size work tape and a $O(\text{poly}(n))$ size certificate read-once tape. This is also the certificate-based definition. (Note that we have a similar definition for NP.)

(I) $\rightarrow$ (II): One can show the simulation of (I) in (II) as follows: (a) use the configuration graph to simulate the computation of (I); (b) use the path in the configuration graph as the certificate in the definition (II). This certificate can be verified using only log-space. (Why?)

(II) $\rightarrow$ (I): A natural approach is to use the non-determinism of (I) to generate all possible certificates. One catch is that we cannot store the whole certificate, which could be of size $O(\text{poly}(n))$, in log-space. To resolve this issue, whenever (II) reads a single bit from the certificate tape, we use the non-deterministic machine to guess a bit. This is a sufficient simulation because one can read the certificate once. (Why? also note that if we allow multiple reads from the certificate tape, then (II) becomes NP.)