Summary of Lecture 2

Reading: [Arora-Barak (AB)] Chap 1.1-1.6; Katz’s Lecture Note 1.

- Definition of the Turing Machine (TM); its components and how the computation is carried out on a TM.
- What kind of computational resources can be defined on TMs? and how are they measured?
- The Church-Turing thesis; strong Church-Turing thesis; extended Church-Turing thesis.
- Robustness of the definition of TMs. How to simulate different variants?
- Universal TMs. How to simulate any another TM on top of a universal TM. For any TM \( M \) running in time \( T(n) \), the simulation of \( M \) a universal TM \( U \) can be done in \( O(T \log T) \). The proof is in [AB] Chap 1.7, using the trick of amortized analysis. This part is not required. The simulation in, e.g., \( O(T^2) \) is, however, much simpler to see.
- Computable vs uncomputable functions. the Halting problem. Note that in the lecture we follow the approach in [AB] Chap 1.5. [AB] Chap 1.5.2 includes an interesting history of the halting problem from the mathematical logic perspective.